THE NEW ENGINEERING

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VENTUNO PRESS

12887 Valewood Drive

Naples, Florida 34119

Library of Congress Control Number: 2002101492

 For information, address the publisher, Ventuno Press, 12887 Valewood Drive, Naples, FL 34119.

ISBN 0-9626220-1-X (paperback)

ISBN 0-9626220-2-8 (hard cover)

for my heroes, especially Marya Sklodowska

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PREFACE

In conventional engineering, problems are solved with the important variables *combined* in contrived parameters such as resistances and coefficients and moduluses. This allows proportional problems to be solved in a simple and direct manner, but generally requires that nonlinear problems be solved in an indirect manner.

In the new engineering, contrived parameters such as resistances and coefficients and moduluses are *abandoned* in order that problems may be solved with the variables *separated*. This allows proportional problems *and* nonlinear problems to be solved in a simple and direct manner. The net result is that the new engineering greatly simplifies the solution of nonlinear problems in general.

History of the new engineering

I used conventional engineering from 1954 until 1963, and the new engineering from 1963 until the present. Of course I have had to use conventional engineering to communicate, but I have used the new engineering to think and to design and to analyze.

In 1963, I accepted a position that placed me in charge of a 300KW boiling liquid metal test facility. In a few months, I recognized that heat transfer coefficients were not a good way to deal with the highly nonlinear behavior of boiling liquid metal. That recognition resulted in the new engineering.

The first publication dealing with the new engineering was published in *Nucleonics* in 1964. It was entitled "New Theory of Thermal Stability in Boiling Systems". Reaction to the article was swift and definitive. *Seven* presumably Ph.D.'s employed at the Argonne National Laboratory wrote to the editor of *Nucleonics* to state:

> *The undersigned, having read "New Theory of Thermal Stability in Boiling Systems", conclude that this article must be either a hoax, or that the paper reviewing procedures followed by Nucleonics are in need of reevaluation.*

Twenty-five years later, the *ASME Journal of Heat Transfer* published a letter from Professor John H. Lienhard that discussed my *Nucleonics* article. The letter states that other workers later duplicated some of my work presented in the article, and they were generally credited in the literature. In summary, the letter states "Many of us have credited an important discovery to the wrong authors".

In 1964, another article dealing with the new engineering was accepted for publication in the *AIChE Journal*. However, my article was never published because the editor received a complaint from a "responsible person". The editor told me it was the only article ever accepted for publication in the *AIChE Journal*, and then not published.

In silent protest, I bought a full page ad in the April 1965 issue of *Nucleonics.* The ad was an abridged version of my article, and offered to send readers free copies of the galley proofs I had received from the *AIChE Journal*. Twenty-seven readers requested and were sent copies.

Thirty years after being accepted for publication in the *AIChE Journal*, the article was published in the *International Journal of the Japanese Society of Mechanical Engineers*. It is entitled "A Critical Examination of the View that Nucleate Boiling Heat-Transfer Data Exhibit Power Law Behavior".

The article's premise is very simple. Literature data generally indicate that nucleate boiling heat flux is *linearly* related to temperature difference. Therefore correlations such as the widely accepted Rohsenow correlation are not rigorous because they describe a highly *nonlinear* relationship.

Over the years, other of my articles on the new engineering have been published in *British Chemical Engineering*, in *Mechanical Engineering*, and in *ASME Journals*.

In 1974, Ventuno Press (of which I am the sole proprietor and sole worker) published *The New Heat Transfer.* It received highly favorable reviews, and highly unfavorable reviews. A Russian translation was published by Mir (Moscow) in 1977. A second edition was published in 1989. The new heat transfer has not yet been widely accepted.

The increasing importance of nonlinear engineering phenomena

When conventional engineering science was conceived several centuries ago, experiment indicated that engineering phenomena generally exhibit proportional behavior. Therefore an engineering science was conceived in which proportional problems could be solved in a simple and direct manner. The fact that nonlinear problems could *not* be solved in a simple and direct manner was of no practical importance.

In the several centuries since conventional engineering was conceived, nonlinear engineering phenomena have become increasingly important:

- Nonlinear electrical devices have enabled instantaneous and world wide communication.
- Nonlinear heat transfer (boiling and condensation) is used in the generation of electricity all over the world.
- Nonlinear (plastic) deformation is increasingly important.

Because of the great and increasing importance of nonlinear engineering phenomena, it is germane to consider whether conventional engineering science should be retained, or whether it should be replaced by an engineering science in which proportional *and* nonlinear problems can be solved in a simple and direct manner.

Conventional engineering

Conventional engineering is based on "laws" that accord with Fourier's view that

> *Engineering phenomena are rigorously described only by equations that are dimensionally homogeneous.*

Based on data obtained by Ohm, Fourier, and Hooke, it was concluded that:

$V \alpha I$	emf is proportional to current	(P-1)
$q \alpha \Delta T$	heat flux is proportional to temperature difference	(P-2)
$\sigma \alpha \varepsilon$	stress is proportional to strain	(P-3)

Expressions (P-1) to (P-3) are inhomogeneous, since the dimensions on the left differ from those on the right. Therefore, in Fourier's view, they are not rigorous. He devised the following method to transform inhomogeneous, proportional expressions into homogeneous equations:

- Convert the proportional expression to an equation by introducing an arbitrary constant.
- Assign a name and a symbol to the arbitrary constant.
- Assign dimensions to the arbitrary constant. Select whatever dimensions make the equation homogeneous.

Fourier's method transforms arbitrary constants into "parameters" that have names, symbols, and dimensions—"parameters" such as electrical resistance R, heat transfer coefficient h, modulus E. These parameters transform inhomogeneous Expressions (P-1) to (P-3) into homogeneous Equations (P-4) to (P-6) generally referred to as "laws"—Ohm's law, "Newton's law of cooling", Young's law:

$$V = I R \tag{P-4}$$

$$q = h \Delta T \tag{P-5}$$

$$\sigma = E \varepsilon \tag{P-6}$$

It is important to note from Eqs. (P-4) to (P-6) that :

- R is the ratio V/I.
- h is the ratio $q/\Delta T$.
- E is the ratio σ/ε.

The problem with ratios such as *R, h,* and *E*

The problem with ratios such as R, h, and E is that they *combine* the important variables. This is mathematically undesirable because non-linear problems are generally much easier to solve if the variables are separated.

Dimensional homogeneity

My view of homogeneity differs considerably from Fourier's view. It results in a new engineering science in which problems are solved with the important variables *separated* rather than combined. The rationale is described in the following:

- Engineering phenomena are cause-and-effect processes. Examples are stress causes strain, electromotive force causes electric current, temperature difference causes heat flux. Since the cause and the effect necessarily have different dimensions, engineering phenomena are inherently *in*homogeneous.
- Since engineering phenomena are inherently *in*homogeneous, there is no foundation for Fourier's view that engineering phenomena are rigorously described only by homogeneous equations. Therefore Fourier's view is rejected.

- Expressions (P-1) to (P-3) do *not* correctly represent the underlying data because they describe impossible relationships. For example, Expression (P-3) states that stress is proportional to strain. But stress cannot be proportional to strain, for the same reason that elephants cannot be proportional to peaches. They are different things, and therefore they cannot be proportional to each other any more than they can be equal to each other.

- Data do *not* indicate how parameters are related to each other. Data indicate how the *numerical values* of parameters are related to each other. For example, Ohm's data indicate that the *numerical value* of electromotive force (in arbitrary dimensions) is proportional to the *numerical value* of electric current (in arbitrary dimensions).

- Expressions (P-1) to (P-3) correctly describe the underlying data only if the symbols represent the *numerical values* of parameters in arbitrary dimensions. For example, Expression (P-1) correctly describes Ohm's data only if V represents the numerical value of electromotive force in arbitrary dimensions, and I represents the numerical value of electric current in arbitrary dimensions.

- Mathematical operations can be performed *only* on numbers—pure numbers, and numbers of things. Mathematical operations cannot be performed on *things* per se. For example, people cannot be divided by airplanes because people and airplanes are *things*. However, the *number* of people can be divided by the *number* of airplanes to determine the average *number* of people per airplane.

- Mathematical operations can *not* be performed on dimensions because dimensions are *things*. For example, feet can not be divided by seconds. If feet could be divided by seconds, it would be possible to answer the question "How many times does a second go into a foot?"

- Because mathematical operations can be performed only on numbers, and because equations involve mathematical operations, valid equations contain only *numbers*, and are *inherently* homogeneous.

- Because valid equations may contain only numbers, parameter symbols in equations must represent *numerical values* of parameters in *specified* dimensions.

- Since valid equations are inherently homogeneous, ratios such as *R*, *h*, and *E* are *unnecessary* because their sole purpose is to make the laws homogeneous.

- Ratios such as *R*, *h*, and *E* are *undesirable* because they make it necessary to solve problems with the variables *combined*, even though nonlinear problems are generally much easier to solve if the variables are *separated*.

- Because ratios such as *R*, *h*, and *E* are unnecessary and undesirable, they and the laws that define them are abandoned. This makes it possible to solve problems with the variables separated.

Principal Differences

The principal differences between conventional engineering and the new engineering are:

- Ratios such as *R*, *h*, and *E* are *abandoned*. In other words, ratios such as V/I, $q/\Delta T$, and σ/ε are *abandoned*.

- Laws that define ratios such as *R, h,* and *E* are *abandoned*.

- Engineering phenomena are described and problems are solved with the variables *separated* rather than *combined* in ratios such as *R, h,* and *E*. For example:

    - Electrical phenomena are described and problems are solved using V and I, but *not V/I—not R—not "resistance"*.

    - Heat transfer phenomena are described and problems are solved using q and ΔT, but *not* $q/\Delta T$*—not h—not "coefficient"*.

    - Stress/strain phenomena are described and problems are solved using σ and ε, but *not* σ/ε*—not E—not "modulus"*.

- Parameter symbols represent the *numerical values* of parameters in *specified* dimensions rather than the parameters themselves.

- Equations are dimensionally homogeneous, but no significance is attached to homogeneity.

Advantages

The principal advantage of the new engineering is that the solution of nonlinear problems in general is greatly simplified because the variables are separated.

A secondary advantage is that the new engineering is easier to learn because problems are solved with the variables separated, the methodology learned and preferred in mathematics. Only in conventional engineering is it standard practice to solve problems with the variables combined.

Scope of this book

This book presents my view of homogeneity, and the new engineering science that results from it. The book also demonstrates the application of the new engineering to the solution of proportional and nonlinear problems that concern electricity, heat transfer, strength of materials, and fluid flow.

Because of my age, this will likely be my last book. But it will not be my last word.

NOMENCLATURE

- Symbols in italics are parameters. For example, *"T"* is temperature.
- Symbols in regular typeface are *numerical values* of parameters in the dimensions specified. For example, "T" is the numerical value of temperature in degrees F.
- ***f*{I}** indicates "function of I".
- **V{I}** and **V = *f*{I}** refer to an equation or graph that describes the relationship between V and I. The symbolism indicates that V and I are separated, and I is the independent variable.
- $\leq_U$ indicates <u>un</u>stable if satisfied.

SYMBOLS

a	arbitrary constant, or numerical value of acceleration in ft/sec^2
a	acceleration
A	numerical value of area in ft^2 (Parts 1, 2, and 4) or in^2 (Part 3)
b	arbitrary constant
c	arbitrary constant
C	q/V (assigned the name electrical "capacitance")
C_p	heat capacity
d	arbitrary constant
D	numerical value of diameter in ft
D	diameter
E	σ/ε (assigned the name material "modulus")
f	friction factor
g	32.2 ft/sec^2

Symbols cont.

g acceleration constant

h *q/ΔT* (assigned the name heat transfer "coefficient")

I numerical value of electric current in amperes

I electric current

k *q/(dT/dx)* (assigned the name thermal "conductivity")

K proportionality constant between q and dT/dx

L length, ft

L *V/(dI/dt)* (assigned the name "electrical inductance"), or length

m arbitrary constant

n arbitrary constant

M *y/x*, mathematical analog of parameters such as *R, h, E*

N dimensionless parameter group identified by subscript

P numerical value of electric power in watts, or pressure in psf, or load in lbs

P electric power or pressure or load

q numerical value of heat flux in Btu/hrft^2, or numerical value of electric charge in amp-secs

q heat flux or electric charge

Q numerical value of heat flow rate in Btu/hr

Q heat flow rate

R *V/I* (assigned the name electrical "resistance")

s numerical value of distance traversed in ft

s distance traversed

Symbols cont.

t	numerical value of time in hours or thickness in feet
t	time or thickness
T	numerical value of temperature in F
T	temperature
U	symbol for $q/\Delta T_{TOTAL}$ (overall heat transfer coefficient)
v	velocity in ft/sec
v	velocity
V	numerical value of emf in volts
V	emf
W	numerical value of fluid flow rate in pps
W	fluid flow rate
x	numerical value of distance in ft
x	distance or arbitrary variable
y	arbitrary variable
β	numerical value of temperature coefficient of volume expansion in F^{-1}
β	temperature coefficient of volume expansion
ε	numerical value of strain (dimensionless) or roughness in feet
ε	strain or roughness
μ	numerical value of absolute viscosity in lbs/ftsec
μ	absolute viscosity
ν	numerical value of kinematic viscosity in ft^2/sec
ν	kinematic viscosity

Symbols cont.

ρ numerical value of density in lbs/ft^3

ρ density

σ numerical value of stress in psi

σ stress

SUBSCRIPTS

CIRC refers to circuit

COMP refers to component

COND refers to conductive

CONV refers to convective

FALL refers to a subsystem in which emf or pressure falls

Gr refers to Grashof number $g\beta\Delta TL^3/\nu^3$

IN refers to a subsystem that includes the heat source

LM refers to log mean

Nu refers to Nusselt number hD/k or equally $qD/\Delta Tk$

OUT refers to a subsystem that includes the heat sink

Pr refers to Prandtl number $C_p\mu/k$

PS refers to power supply

Re refers to Reynolds number DG/μ

RISE refers to a subsystem in which emf or pressure rises

SINK refers to heat sink

SOURCE refers to heat source

W refers to wall

Chapter 1

Conventional engineering and new engineering

1 Introduction

Engineering phenomena are cause-and-effect processes. Parameters that identify causes and effects are primary parameters. For example:

- Electromotive force V causes electric current I.
- Temperature difference ΔT causes heat flux q.
- Stress σ causes strain ε.

The principal difference between conventional engineering and the new engineering is the manner in which the primary parameters are used.`

- In conventional engineering, the primary parameters in each discipline are *combined* and *implicit* in a ratio that has a name and a symbol. The primary parameters *and* their ratio are used to describe phenomena, and to solve problems.
- In the new engineering, the primary parameters in each discipline are *not* combined. They remain *separate* and *explicit*. The primary parameters *without their ratio* are used to describe phenomena, and to solve problems.

For example, in conventional engineering:

- Electrical phenomena are described and problems are solved using V and I *and* V/I. The ratio V/I is electrical resistance, symbol R.
- Heat transfer phenomena are described and problems are solved using q and ΔT *and* $q/\Delta T$. The ratio $q/\Delta T$ is heat transfer coefficient, symbol h.
- Stress/strain phenomena are described and problems are solved using σ and ε *and* σ/ε. The ratio σ/ε is material modulus, symbol E.

In the new engineering:

- Electrical phenomena are described and problems are solved using V and I. *Not used* are V/I, R, and the word "resistance".

- Heat transfer phenomena are described and problems are solved using q and ΔT. *Not used* are $q/\Delta T$, h, and the word "coefficient".

- Stress/strain phenomena are described and problems are solved using σ and ε. *Not used* are σ/ε, E, and the word "modulus".

The principal advantage of the new engineering is that nonlinear problems in general are much easier to solve because the primary parameters in each discipline are separate and explicit—ie they are *not* combined and implicit in a ratio that has a name and a symbol.

The simplification results because, if a problem concerns nonlinear behavior, the ratio of primary parameters is *variable*. If this variable ratio is used in the analysis of a problem, the analysis usually must be indirect. If it is *not* used, the analysis is direct and *much* simpler.

Aside from the manner in which primary parameters are used, the only other important differences between conventional engineering and the new engineering concern symbolism and dimensional homogeneity.

- In conventional engineering, symbols represent parameters. Only homogeneous equations are considered scientifically rigorous.

- In the new engineering, symbols represent the numerical values of parameters in specified dimensions. Equations are inherently homogeneous because they contain only numbers. However, homogeneity is of no significance. Equations are considered rigorous if they accurately describe the behavior they purport to describe.

The new engineering is easy to learn because it uses only parameters also used in conventional engineering, and because solving problems with the variables separated is the methodology learned and preferred in pure mathematics. Only in conventional engineering is it standard practice to solve problems with the variables combined.

This book describes the new engineering, and demonstrates its application to the solution of proportional and nonlinear problems that concern electricity, heat transfer, strength of materials, and fluid flow.

1.1 Conventional engineering

In conventional engineering, laws combine the primary parameters in ratios that are assigned symbols and names:

- Ohms law, Eq. (1-1), combines V and I in the ratio V/I. This ratio is assigned the symbol R and the name "resistance".

  $$R = V/I \tag{1-1}$$

  In words of the great Clerk Maxwell (1873):

  (Ohm's law states that) the resistance of a conductor . . . is defined to be the ratio of the electromotive force to the strength of the current which it produces.

  Similarly, the Encyclopedia Brittanica (1999-2000) states:

  Precisely, $R = V/I$

- "Newton's law of cooling", Eq. (1-2), combines q and ΔT in the ratio $q/\Delta T$. This ratio is assigned the symbol h and the name "coefficient".

  $$h = q/\Delta T \tag{1-2}$$

- Young's law, Eq. (1-3), combines σ and ε.in the ratio σ/ε. This ratio is assigned the symbol E and the name "modulus".

  $$E = \sigma/\varepsilon \tag{1-3}$$

Note that:

- If a problem concerns proportional phenomena, ratios such as R, h, and E are *constants* in the analysis.

- If a problem concerns nonlinear phenomena, ratio such as R, h, and E are *variables* in the analysis.

If a problem concerns proportional phenomena, the solution is simple and direct based on ratios such as R, h, and E because they are *constants* in the analysis.

However, if a problem concerns nonlinear phenomena, the solution must usually be *indirect* because ratios such as R, h, and E are *variables* in the analysis.

1.2 The mathematical analog of *R, h,* and *E*

Eq. (1-4) is the mathematical analog of Eqs. (1-1) to (1-3).

$$M = y/x \tag{1-4}$$

Note the following:

- y/x is the mathematical analog of V/I, $q/\Delta T$, and σ/ε.
- M is the mathematical analog of R, h, and E. Note that M is y/x, R is V/I, h is $q/\Delta T$, and E is σ/ε.
- M is a *constant* if y is proportional to x, just as R is a constant if V is proportional to I, and similarly for h and E.
- M is a *variable* if y is *not* proportional to x, just as R is a variable if V is not proportional to I, and similarly for h and E.
- Mathematics has *no use* for M because it generally complicates the solution of nonlinear equations by making it necessary to solve them in an indirect manner.
- The new engineering has *no use* for R, h, and E for the same reason that mathematics has no use for M.

1.3 The importance of separating the variables

The following example illustrates the importance of eliminating ratios such as M, R, h, E in order to separate the variables. Note in the example that:

- y/x is the mathematical analog of V/I, $q/\Delta T$, and σ/ε.
- The ratio M is the mathematical analog of the ratios R, h, and E.
- x and y can be separated only if M is eliminated.
- If M is *not* eliminated, an *indirect* solution is required.
- If M is eliminated, a direct and *much* simpler solution is possible.

Problem 1.3 ***(to be solved by the reader)***
Without eliminating M, solve Eq. (1-5) for x, given that M is the symbol for y/x, and $y = 2.7$.

$$M = 2 + y + 5/x \tag{1-5}$$

Because the problem statement does not allow M (the symbol for y/x) to be eliminated, the problem cannot be solved in a direct manner by simply substituting 2.7 for y. The problem must be solved in an indirect manner such as the following:

- Estimate an initial value of x.
- Substitute the estimated x in Eq. (1-5) to obtain an estimate of M.
- Substitute the estimated M in the equation $M = 2.7/x$ to obtain a second estimate of x.
- Iterate until the solution converges.
- If the solution does not converge, use a more powerful iteration method. Or solve the problem graphically, or by trial-and-error.

In mathematics, the separation of variables is so routine that the solution of Eq. (1-5) with x and y combined in M seems bizarre. Given a choice, *every* reader would solve Eq. (1-5) by first *eliminating M* in order to separate x and y. When M is eliminated, Eq. (1-6) results.

$$y = (2x + 5)/(1-x) \tag{1-6}$$

With x and y separated in Eq. (1-6), the value of x is determined simply and directly by substituting 2.7 for y. The answer to Problem 1.3 is $x = -.489$ at $y = 2.7$.

In conventional engineering, problems are necessarily solved with the variables combined because R is the ratio V/I, h is the ratio $q/\Delta T$, and E is the ratio σ/ε, just as M is the ratio x/y.

In mathematics and in the new engineering, problems are solved with the variables *separated* because ratios that combine the variables (such as M, R, h, E) are *not used*. The end result is that the new engineering greatly simplifies the solution of nonlinear problems in general.

1.4 Parameter symbols in the new engineering

In conventional engineering, parameter symbols represent parameters. For example, *T* and *I* might be defined in a text nomenclature as follows:

- *T* = temperature
- *I* = electric current

In the new engineering, a parameter symbol represents the *numerical value* of a parameter in a *specified* dimension. *Any* dimensions may be used. The only requirement is that they be specified. For example, the symbols T and I might be defined in a text nomenclature as follows:

- T = *numerical value* of temperature *in degrees Fahrenheit*
- I = *numerical value* of electric current *in amperes*

In order to distinguish between the two types of symbols, those in italics represent parameters of unspecified dimension, and those in regular typeface represent the numerical values of parameters in dimensions specified in the Nomenclature. For example, "*T*" is temperature, whereas "T" is the numerical value of temperature in degrees Fahrenheit.

Thus "T = 23" states "the numerical value of the temperature in degrees Fahrenheit equals 23" or equally "temperature in degrees F equals 23". The expression "T = 23 degrees Fahrenheit" is unacceptable because the symbol specifies the dimension, and therefore "Fahrenheit" is redundant.

1.5 Equations in the new engineering

In the new engineering:

- Parameter symbols represent the *numerical values* of parameters in specified dimensions. Therefore equations contain only numbers.
- Because equations contain only numbers, they are inherently homogeneous. However, no significance is attached to homogeneity.

Because symbols represent numerical values of parameters rather than parameters, equations are interpreted differently than in conventional engineering. For example, the Nomenclature indicates that Eq. (1-7) is to be interpreted in either of the following equivalent ways:

- The numerical value of heat flux in $Btu/hrft^2$ equals 4.6 times the numerical value of temperature difference in degrees F raised to the 1.33 power.

- The heat flux in $Btu/hrft^2$ is numerically equal to 4.6 times the temperature difference in degrees F raised to the 1.33 power.

$$q = 4.6\ \Delta T^{1.33} \tag{1-7}$$

1.6 Inhomogeneous equations in the twentieth century

The manner in which equations are interpreted in the new engineering closely resembles the manner in which inhomogeneous equations were interpreted when they were commonly used several decades ago.

For example, Perry (1950) recommends the following equation for heat loss from horizontal pipes to air at atmospheric pressure and normal temperatures:

$$h = 0.5\ (\Delta T/D)^{0.25}$$

The Nomenclature in Perry (1950) indicates that *h* is in $Btu/hrft^2F$, *T* is in degrees F, and *D* is in inches. The equation is interpreted as follows:

> The heat transfer coefficient in $Btu/hrft^2F$ is numerically equal to 0.5 times (temperature difference in degrees F divided by diameter in inches) raised to the 0.25 power.

Note that the distinguishing features in the above interpretation are essentially identical to distinguishing features in the new engineering:

- Parameter symbols identify parameters *and* dimensions.

- Mathematical operations are performed *only on the numerical values* of dimensioned quantities.

1.7 Describing engineering phenomena in the new engineering

In the new engineering, the *behavior* of the primary parameters is used to describe engineering phenomena. No significance is attached to the *ratio* of the primary parameters (such as the resistance or the coefficient or the modulus). For example:

- Resistive electrical behavior is an equation or chart in the form V{I} or I{V}—ie in the form V = *f*{I} or I = *f*{V}. (The symbolism indicates that V and I are *separate* and *explicit*.)

  Eqs. (1-8) and (1-9) and Figure (1-1) are in behavior form. Eqs. (1-8R) and (1-9R) and Figure (1-1R) are identical expressions in resistance form—ie in *V/I* (symbol *R*) form.

  $$V = 3.4\ I \qquad (1\text{-}8)$$

  $$V/I = R = 3.4 \text{ ohms} \qquad (1\text{-}8R)$$

  $$V = 6.5\ I^{1.6} \qquad (1\text{-}9)$$

  $$V/I = R = 6.5\ I^{.6} \text{ ohms} \qquad (1\text{-}9R)$$

- Convective heat transfer behavior is an equation or chart in the form q{ΔT} or ΔT{q}—ie in the form q = *f*{ΔT} or ΔT = *f*{q}.

  Eqs. (1-10) to (1-12) are in behavior form. Eqs. (1-10C) to (1-12C) are identical equations in coefficient form—ie in *q/ΔT* (symbol *h*) form.

  $$q = .023\ (k/D)\ N_{Re}^{.8}\ N_{Pr}^{.4}\ \Delta T \qquad (1\text{-}10)$$

  $$q/\Delta T = h = .023\ (k/D)\ N_{Re}^{.8}\ N_{Pr}^{.4}\ \text{Btu/hrft}^2\text{F} \qquad (1\text{-}10C)$$

  $$q = 148\ \Delta T \qquad (1\text{-}11)$$

  $$q/\Delta T = h = 148\ \text{Btu/hrft}^2\text{F} \qquad (1\text{-}11C)$$

  $$q = 15\ \Delta T^{1.33} \qquad (1\text{-}12)$$

  $$q/\Delta T = h = 15\ \Delta T^{.33}\ \text{Btu/hrft}^2\text{F} \qquad (1\text{-}12C)$$

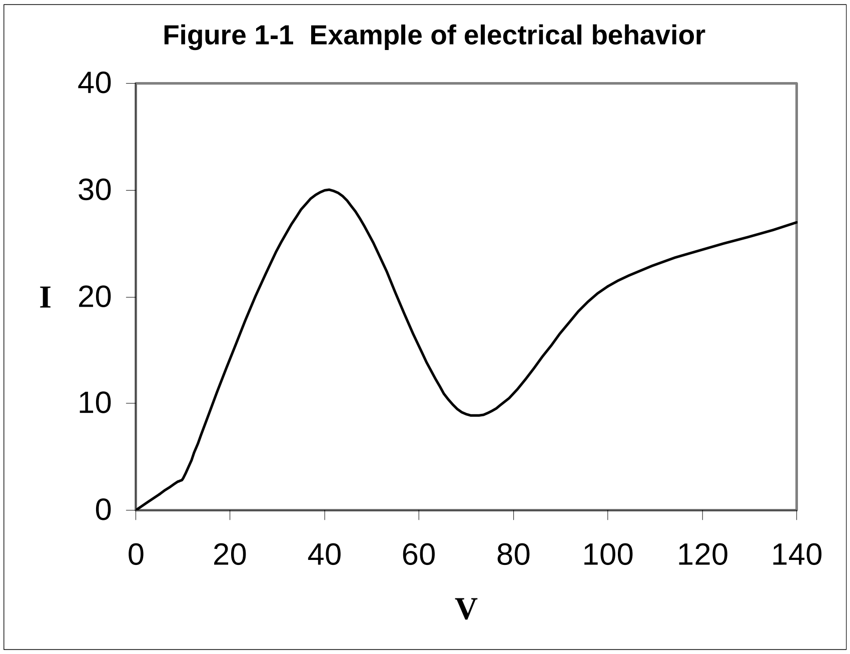
Figure 1-1 Example of electrical behavior
40
30
I
20
10
0
0
20
40
60
80
100
120
140
V

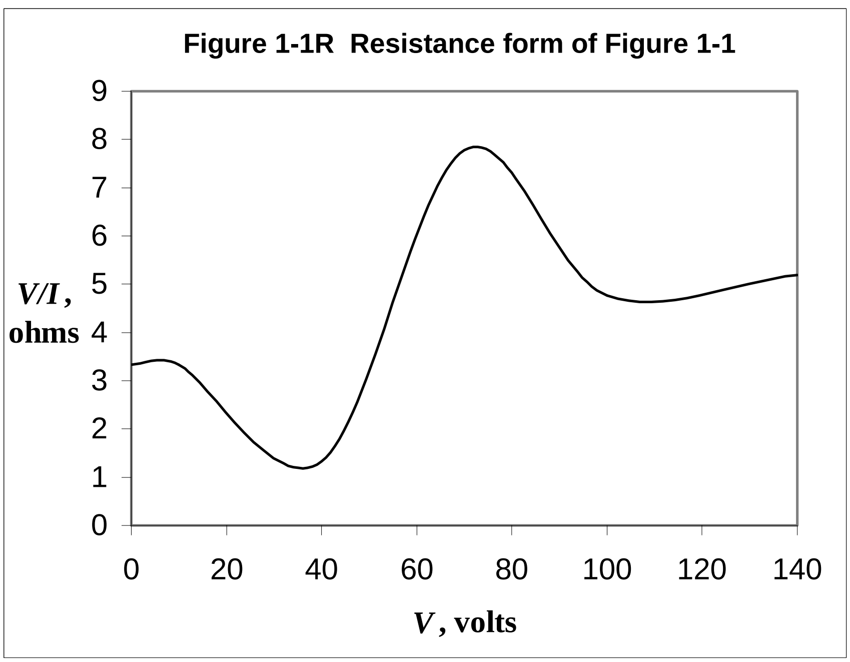
Figure 1-1R Resistance form of Figure 1-1
9
8
7
6
V/I,
ohms
5
4
3
2
1
0
0
20
40
60
80
100
120
140
V, volts

- Stress/strain behavior is an equation or chart in the form $\sigma\{\varepsilon\}$ or $\varepsilon\{\sigma\}$—ie in the form $\sigma = f\{\varepsilon\}$ or $\varepsilon = f\{\sigma\}$. (Stress/strain charts used in conventional engineering are in behavior form.)

  Eqs. (1-13) and (1-14) are in behavior form. Eqs. (1-13M) and (1-14M) are identical equations in modulus form—ie in σ/ε (symbol E) form.

$$\sigma = 30 \times 10^6\ \varepsilon \qquad (1\text{-}13)$$

$$\sigma/\varepsilon = E = 30 \times 10^6 \ psi \qquad (1\text{-}13M)$$

$$\sigma = 1.55 \times 10^6\ \varepsilon^{.7} \qquad (1\text{-}14)$$

$$\sigma/\varepsilon = E = 1.55 \times 10^6 \varepsilon^{-.3}\ psi \qquad (1\text{-}14M)$$

1.8 Solving problems in the new engineering

In the new engineering, problems are solved with the primary parameters separate, just as in mathematics, problems are solved with the variables separate. In other words:

- Electrical problems are solved using V and I. The ratio *V/I* (symbol *R*) is *not used*.

- Heat transfer problems are solved using q and ΔT. The ratio *q/ΔT* (symbol *h*) is *not used*.

- Stress/strain problems are solved using σ and ε. The ratio *σ/ε* (symbol *E*) is *not used*.

- Mathematical problems are solved using *x* and *y*. The ratio *y/x* (symbol *M*) is *not used*.

The particular advantage of separating the primary parameters is that it greatly simplifies the solution of nonlinear problems by making it possible to solve them in a direct manner. If the primary parameters are combined in ratios such as *R, h,* and *E*, nonlinear problems must usually be solved in an indirect, unnecessarily difficult manner.

1.9 The conventional view of dimensional homogeneity[1]

In conventional engineering, it is implicitly assumed that engineering phenomena exhibit homogeneous behavior. Therefore in the conventional view, scientific rigor demands that engineering phenomena be described by equations that are also homogeneous.

(The conventional view of dimensional homogeneity was conceived by Fourier (1822). Earlier scientists, such as Newton and his contemporaries, generally used inhomogeneous expressions.)

The manner in which Young's law is obtained from Hooke's law reflects the conventional view of homogeneity. Hooke's law, Expression (1-15), states that stress is proportional to strain. Note that it is inhomogeneous because stress and strain have different dimensions.

$$\sigma \, \alpha \, \varepsilon \qquad (1\text{-}15)$$

The inhomogeneous Hooke's law is transformed to the homogeneous Young's law in the following way:

- Convert Expression (1-15) to an equation by introducing an arbitrary constant.
- Assign the name "modulus" and the symbol E to the constant.
- Assign dimensions to the constant. Assign dimensions that make the equation homogeneous. (Since strain is dimensionless, the equation will be homogeneous if E is assigned the dimension of stress.)

Eq. (1-16), the so-called Young's law, is the result of the transformation.

$$\sigma = E \, \varepsilon \qquad (1\text{-}16)$$

Other homogeneous laws, such as "Newton's law of cooling" and Ohm's law, are also generated in the above manner.

Note that Eq. (1-16) is homogeneous, and that E is the ratio σ/ε. Also note that this ratio is constant if σ is proportional to ε, and variable if σ is not proportional to ε.

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[1] Chapter 17 presents a more comprehensive discussion of the conventional engineering view of dimensional homogeneity.

1.10 The new engineering view of engineering equations

Hooke's experiment did *not* demonstrate that stress is proportional to strain. Stress and strain are *different things*. They cannot be proportional to each other any more than they can be equal to each other.

Hooke's data actually demonstrated that:

> The *numerical value* of stress in arbitrary dimensions is proportional to the *numerical value* of strain.

Expression (1-17) is the correct symbolic description of Hooke's empirical conclusion:

$$\sigma \, \alpha \, \varepsilon \qquad (1\text{-}17)$$

In Expression (1-17), σ represents the *numerical value* of stress in arbitrary dimensions, and ε represents the *numerical value* of strain (which has no dimensions). Note that Expression (1-17) is valid with the stress in arbitrary dimensions because proportional expressions are qualitative. When Expression (1-17) is converted to equation form, the stress dimension must be made specific.

Expression (1-17) is homogeneous because it contains only numbers. Therefore its conversion to a homogeneous equation requires merely the introduction of an arbitrary constant. It does *not* require the introduction of *E*—the ratio of the primary parameters σ and ε.

Similarly, Ohm's experiment did not indicate that emf is proportional to current. It indicated that the numerical value of emf in arbitrary dimensions is proportional to the numerical value of current in arbitrary dimensions. Since this expression of proportionality is homogeneous, its conversion to a homogeneous equation requires merely the introduction of an arbitrary constant. It does *not* require the introduction of *R*—the ratio of the primary parameters emf and current.

Nor does homogeneity require the introduction of *h*—the ratio of the primary parameters *q* and ΔT.

In short, engineering equations do not describe how parameters are related. They describe how the *numerical values* of parameters are related. Therefore they are inherently homogeneous, and do *not* require the introduction of ratios such as *R, h,* and *E* in order to achieve homogeneity.

1.11 The new engineering view of homogeneity[2]

The new engineering view of homogeneity is:

- Engineering phenomena are cause and effect processes. Stress causes strain, temperature difference causes heat flux, electromotive force causes electric current.

  Since the dimension of each effect necessarily differs from the dimension of the corresponding cause, engineering phenomena exhibit *in*homogeneous behavior.

- Since engineering phenomena generally exhibit *in*homogeneous behavior, there is no foundation for Fourier's view that phenomena are rigorously described only by equations that are homogeneous. Therefore Fourier's view is rejected.

- Scientific rigor has nothing to do with homogeneity. Scientific rigor requires that equations accurately describe the behavior of the engineering phenomena they purport to describe.

- Engineering equations describe how the *numerical values* of parameters are related. Therefore they are inherently homogeneous.

- Equations properly contain only numbers. Therefore symbols in engineering equations must represent numerical values of parameters in specified dimensions.

1.12 Principal differences

The new engineering differs from conventional engineering in the following ways:

- Ratios of primary parameters such as R, h, and E are *not used*.

- Engineering phenomena are described and problems are solved with the primary parameters separate and explicit, rather than combined and implicit in ratios such as R, h, and E.

- The focus is on the *behavior* of the primary parameters rather than the *ratio* of the primary parameters.

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[2] Chapter 18 presents a more comprehensive discussion of the new engineering view of dimensional homogeneity.

- Parameter symbols represent the *numerical values* of parameters in specified dimensions, rather than the parameters themselves in unspecified dimensions.
- Equations are inherently homogeneous because they contain only numbers, rather than because they contain dimensioned ratios such as R, h, and E.
- Homogeneity is considered to have no significance, rather than being considered essential for scientific rigor.

1.13 Advantages of the new engineering

The principal advantage of the new engineering is that it greatly simplifies the solution of nonlinear problems in general.

The simplification results because the primary parameters are separated in the new engineering, and this makes it possible to solve nonlinear problems in a direct manner. In conventional engineering, the primary parameters are combined in ratios such as V/I (symbol R), $q/\Delta T$ (symbol h), and σ/ε (symbol E), and this generally makes it necessary to solve nonlinear problems in an indirect, unnecessarily difficult manner.

Other advantages of the new engineering are:

- Problems are solved using methodology learned in mathematics—ie problems are solved with the variables separated. Only in conventional engineering is it standard practice to solve problems with the variables combined.
- It is more logical. For example, it is logical to solve problems that concern V and I using only the variables V and I. It is *not* logical to solve problems that concern V and I using the variables V and I *and* the ratio V/I (symbol R) that may also be variable.
- There is less to learn. For example, it is not necessary to learn how to solve problems using ratios such as V/I (symbol R), $q/\Delta T$ (symbol h), or σ/ε (symbol E) because they are not used in the new engineering.

Chapter 2

Example problems that illustrate electrical analysis using behavior methodology

2 Introduction

This chapter contains example problems that illustrate the analysis of resistive electrical components and systems using "behavior" methodology—ie methodology in which V and I are *separate* and *explicit*. The problems include proportional components and nonlinear components, and demonstrate that the analysis of electrical components and systems is simple and direct using behavior methodology.

2.1 The purpose of the example problems in Chapters 2 and 3

The problems in this chapter are stated in behavior form, and are solved using behavior methodology. The problems include proportional and nonlinear electrical problems that deal with individual components, and with systems.

The problems in this chapter are restated in Chapter 3 using resistance terminology. The reader is requested to solve the problems using resistance methodology in order to experience the simplification that results from behavior methodology. In Chapters 2 and 3, note that:

- Problems 2.5/1, 2.6/1, and 2.6/4 concern proportional circuits. They can be solved in a simple and direct manner using either behavior methodology or resistance methodology.

- Problems 2.5/2 and 2.6/2 concern very simple nonlinear circuits. They can be solved in a simple and direct manner using either behavior methodology or resistance methodology.

- Problems 2.5/3, 2.6/3, 2.6/5, and 2.6/6 involve more complex nonlinear circuits. They can be solved in a simple and direct manner using behavior methodology, but must be solved in an indirect and much more difficult manner using resistance methodology.

2.2 Electrical component analysis using behavior methodology

In the new engineering, problems that concern electrical components are solved using "behavior" methodology. If a problem concerns a resistive electrical component, behavior methodology is described by the following:

- The problem statement specifies the value of V or I applied to the component.

- The electrical behavior of the component is given in the form $V_{COMP}\{I_{COMP}\}$ or $I_{COMP}\{V_{COMP}\}$. In other words, Eq. (2-1) or Eq. (2-2) is given in analytical or graphical form.

$$V_{COMP} = f\{I_{COMP}\} \quad (2\text{-}1)$$

$$I_{COMP} = f\{V_{COMP}\} \quad (2\text{-}2)$$

(Note that Eqs. (2-1) and (2-2) have nothing to do with the resistance defined by Ohm's law, Eq. (2-3).)

$$R = V/I \quad (2\text{-}3)$$

- If the value of I_{COMP} is specified, V_{COMP} is determined from Eq. (2-1) or (2-2), and similarly if V_{COMP} is specified.

- The electric power dissipated in the component is determined from Eq. (2-4).

$$P_{COMP} = V_{COMP}\, I_{COMP} \quad (2\text{-}4)$$

The examples in Section 2.5 illustrate how behavior methodology is used to solve problems that concern a single electrical component.

2.3 Electrical system analysis using behavior methodology

If a problem concerns an electrical system that consists of a power supply and a circuit in which there are several resistive electrical components, the behavior methodology of the new engineering is described by the following:

- The problem statement provides a drawing of the electric circuit, and usually specifies the power supply voltage (V_{PS}) or voltage operating range. (V_{PS} may be specified as a function of I_{PS}, or I_{PS} may be specified instead of V_{PS}). The solution of the problem requires that the distribution of emf and electric current be determined throughout the circuit or in part of the circuit.

- The electrical behavior of each component in the circuit is given in the form $V_{COMP}\{I_{COMP}\}$, or the form $I_{COMP}\{V_{COMP}\}$.

- Inspect the circuit diagram and write circuit behavior equations (ie equations in the form V{I} or I{V}) by noting that:

  - When electrical components are connected in series, the emf's are additive, and the electric currents are equal.

  - When electrical components are connected in parallel, the emf's are equal, and the electric currents are additive.

- Determine I_{CIRC} from component behavior equations or charts, and circuit behavior equations.

- For each component, determine V_{COMP} and I_{COMP} from V_{PS}, I_{CIRC}, the component behavior equations, and the circuit behavior equations.

- Determine the electric power dissipated in each component from Eq. (2-4).

The examples in Section 2.6 illustrate how behavior methodology is used to solve problems that concern series circuits, and series-parallel circuits.

2.4 A preview of the problems

Example problems 2.5/1 through 2.5/3 demonstrate the analysis of resistive electrical components using behavior methodology:

- The component in Problem 2.5/1 exhibits proportional behavior.

- The component in Problem 2.5/2 exhibits moderately nonlinear behavior.

- The component in Problem 2.5/3 exhibits highly nonlinear behavior.

Example problems 2.6/1 to 2.6/6 demonstrate the analysis of electrical systems using behavior methodology.

- Problem 2.6/1 concerns analysis of a series connected circuit in which all components exhibit proportional behavior. Notice that the problem is to calculate the value of the current, and the value of the current is calculated.

  (Using resistance methodology, if the problem is to calculate the value of the current, the value of the "overall resistance" is calculated first, and then the overall resistance is used to calculate the value of the current.)

- Problem 2.6/2 concerns analysis of a series connected circuit in which one component exhibits nonlinear behavior. Note that the analysis differs from the analysis in Problem 2.6/1 only in that $I_{CIRC}\{V_{CIRC}\}$ is a proportional equation in Problem 2.6/1, and a nonlinear equation in Problem 2.6/2.

- Problem 2.6/3 concerns analysis of a series connected circuit in which one of the components exhibits highly nonlinear behavior that is described graphically. Note that the analysis differs from the analysis in Problem 2.6/2 only in that the analysis is performed graphically rather than analytically.

- Problems 2.6/4 to 2.6/6 differ from Problems 2.6/1 to 2.6/3 in that they concern series-parallel connected circuits instead of series connected circuits.

Notice that all the problems are solved in a simple and direct manner using behavior methodology.

2.5 Example problems—Analysis of electrical components

<u>Problem 2.5/1</u>

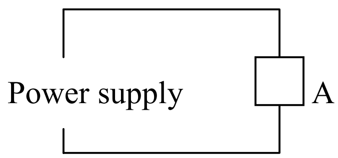
Problem statement

In Figure (2-1), what power supply emf would cause a current of 7.2 amperes? What power would be dissipated in Component A?

Given

The electrical behavior of Component A is given by Eq. (2-5).

$$V_A = 5.6\ I_A \tag{2-5}$$



<u>Figure 2-1</u> Electric system, Problems 2.5/1 to 2.5/3

Analysis

- Substitute the specified value of I_A in Eq. (2-5):

$$V_A = 5.6\ (7.2) = 40.3 \tag{2-6}$$

- Substitute in Eq. (2-4):

$$P_A = V_A\, I_A = 40.3(7.2) = 290 \tag{2-7}$$

Solution

An emf of 40.3 volts would cause a current of 7.2 amps in Component A. The power dissipated in Component A would be 290 watts.

Problem 2.5/2

Problem statement

In Figure (2-1), what current would be caused by a power supply emf of 75 volts? What power would be dissipated in Component A?

Given

The electrical behavior of Component A is given by Eq. (2-8).

$$V_A = 4.7\ I_A^{1.4} \tag{2-8}$$

Analysis

- Substitute the specified value of V_A in Eq. (2-8):

$$75 = 4.7\ I_A^{1.4} \tag{2-9}$$

- Solve Eq. (2-9), and obtain $I_A = 7.23$.

- Substitute in Eq. (2-4):

$$P_A = V_A\, I_A = 75(7.23) = 542 \tag{2-10}$$

Solution

In Figure (2-1), a current of 7.23 amps would be caused by a power supply emf of 75 volts. The power dissipated in Component A would be 542 watts.

Problem 2.5/3

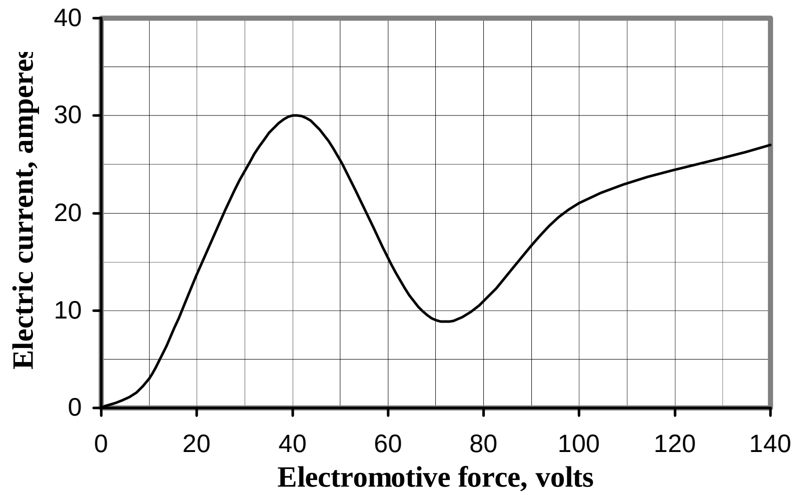
Problem statement

In Figure (2-1), what power supply emf would cause a current of 20 amps? What power would be dissipated in Component A?

Given

The electrical behavior of Component A is given by Figure (2-2).

<u>Figure 2-2</u> **Electrical behavior of Component A, Problem 2.5/3**



Analysis

- Inspect Figure (2-2) and note that a current of 20 amps would result from a power supply emf of 26, 55, or 97 volts.

- Substitute in Eq. (2-4):

$$P_A = V_A I_A \tag{2-11}$$

$$P_A = 20(26) \text{ or } 20(55) \text{ or } 20(97) \tag{2-12}$$

Solution

In Figure (2-1), a current of 20 amps would be caused by an emf of 26 or 55 or 97 volts. The power dissipated in Component A would be 520 or 1100 or 1940 watts. The information given is not sufficient to determine a unique solution.

2.6 Example problems—Analysis of electrical systems

Problem 2.6/1

Problem statement
What are the values of emf, electric current, and electric power for each component in Figure (2-3)?

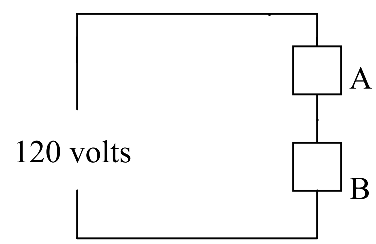


Figure 2-3 Electric system in Problem 2.6/1

Given
The electrical behavior of Components A and B is given by Eqs. (2-13) and (2-14).

$$V_A = 17\ I_A \qquad (2\text{-}13)$$

$$V_B = 9.4\ I_B \qquad (2\text{-}14)$$

Analysis
- Inspect Figure (2-3) and note that, since Components A and B are connected in series, their emf values are additive, and their electric currents are equal.

$$V_A + V_B = V_{CIRC} \qquad (2\text{-}15)$$

$$I_A = I_B = I_{CIRC} \qquad (2\text{-}16)$$

<u>Problem 2.6/1</u> cont.

- Determine $I_{CIRC}\{V_{CIRC}\}$ by combining Eqs. (2-13) to (2-15), and using Eq. (2-16).

$$17\ I_{CIRC} + 9.4\ I_{CIRC} = V_{CIRC} \tag{2-17}$$

- Solve Eq. (2-17) for $V_{CIRC} = 120$, and obtain $I_{CIRC} = 4.55$.

- Determine I_A and I_B from Eq. (2-16).

- Substitute I_A and I_B in Eqs. (2-13) and (2-14):

$$V_A = 17\ (4.55) = 77.3 \tag{2-18}$$

$$V_B = 9.4\ (4.55) = 42.8 \tag{2-19}$$

- Substitute in Eq. (2-4):

$$P_A = V_A\ I_A = 77.3(4.55) = 352 \tag{2-20}$$

$$P_B = V_B\ I_B = 42.8(4.55) = 195 \tag{2-21}$$

Solution

For Component A, the values of emf, electric current, and electric power are 77.3 volts, 4.55 amps, and 352 watts. For Component B, the values are 42.8 volts, 4.55 amps, and 195 watts.

Problem 2.6/2

Problem statement

What are the values of emf, electric current, and electric power for each component in Figure (2-4)?

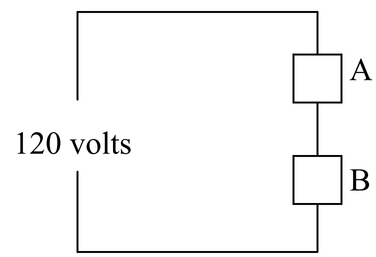


Figure 2-4 **Electric system in Problem 2.6/2**

Given

The electrical behavior of Components A and B is given by Eqs. (2-22) and (2-23):

$$V_A = 3.6\ I_A \qquad (2\text{-}22)$$

$$V_B = 4.8\ I_B^{1.5} \qquad (2\text{-}23)$$

Analysis

- Inspect Figure (2-4) and note that Components A and B are connected in series. Therefore their emf values are additive, and their electric currents are equal.

$$V_A + V_B = V_{CIRC} \qquad (2\text{-}24)$$

$$I_A = I_B = I_{CIRC} \qquad (2\text{-}25)$$

<u>Problem 2.6/2</u> cont.

- Determine $I_{CIRC}\{V_{CIRC}\}$ by combining Eqs. (2-22) to (2-24), and using Eq. (2-25):

$$3.6\ I_{CIRC} + 4.8\ I_{CIRC}^{1.5} = V_{CIRC} \qquad (2\text{-}26)$$

- Solve Eq. (2-26) for $V_{CIRC} = 120$, and obtain $I_{CIRC} = 7.26$.

- Determine I_A and I_B from Eq. (2-25).

- Substitute in Eqs. (2-22) and (2-23):

$$V_A = 3.6(7.26) = 26 \qquad (2\text{-}27)$$

$$V_B = 4.8(7.26)^{1.5} = 94 \qquad (2\text{-}28)$$

- Substitute in Eq. (2-4):

$$P_A = V_A I_A = 26(7.26) = 189 \qquad (2\text{-}29)$$

$$P_B = V_B I_B = 94(7.26) = 682 \qquad (2\text{-}30)$$

Solution

For Component A, the values of emf, electric current, and electric power are 26 volts, 7.26 amperes, and 189 watts. For Component B, the values are 94 volts, 7.26 amperes, and 682 watts.

Problem 2.6/3

Problem statement
What are the values of emf, electric current, and electric power for each component in Figure (2-5)?

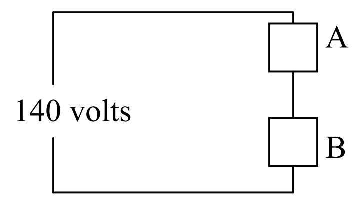
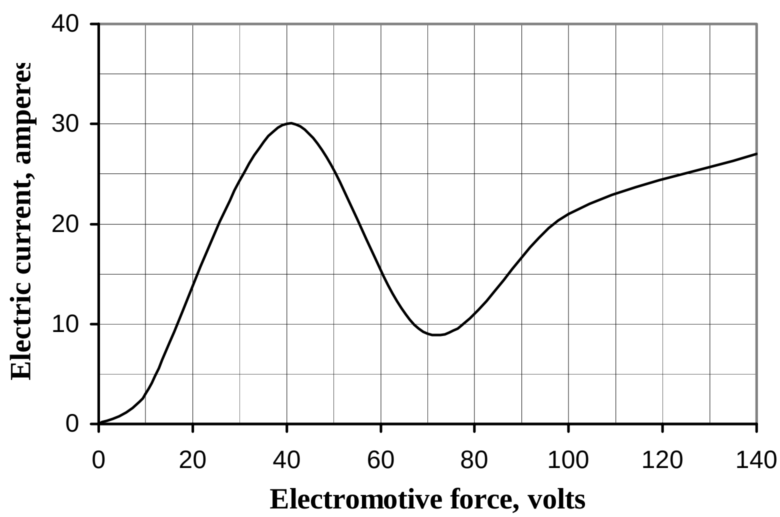


Figure 2-5 **Electric system in Problem 2.6/3**

Given
The electrical behavior of Component A is given by Eq. (2-31). The electrical behavior of Component B is given by Figure (2-6).

$$V_A = 3.89\ I_A \qquad (2\text{-}31)$$

Figure 2-6 **Electrical behavior of Component B, Problem 2.6/3**



<u>Problem 2.6/3</u> cont.

Analysis

- Inspect Figure (2-5) and note that:

$$V_A + V_B = V_{CIRC} \tag{2-32}$$

$$I_A = I_B = I_{CIRC} \tag{2-33}$$

- Determine $I_{CIRC}\{V_{CIRC}\}$ over a range that includes 140 volts:
    - Select (I_B, V_B) coordinates from Figure (2-6).
    - At each (I_B, V_B) coordinate, use Eqs. (2-31) and (2-33) to calculate $V_A(I_B)$.
    - Use Eq. (2-32) to calculate V_{CIRC}.
    - The calculated (V_{CIRC}, I_{CIRC}) coordinates are in Table (2-1).
    - Plot the $I_{CIRC}\{V_{CIRC}\}$ coordinates from Table (2-1). The plotted range must include $V_{CIRC} = 140$. The plot is Figure (2-7).

- Note in Figure (2-7) that there are 3 possible solutions for $I_{CIRC}\{V_{CIRC} = 140\}$. The solutions are 14, 22, and 27 amperes.

- Substitute in Eq. (2-33) to determine I_A and I_B.

$$I_A = I_B = I_{CIRC} = 14 \text{ or } 22 \text{ or } 27$$

- Substitute in Eq. (2-31) to determine V_A.

$$V_A = 3.89\, I_A = 3.89(14 \text{ or } 22 \text{ or } 27)$$

- Substitute in Eq. (2-32) to determine V_B.

$$V_B = V_{CIRC} - V_A = 140 - V_A \tag{2-34}$$

- Substitute in Eq. (2-4) to determine P_A and P_B.

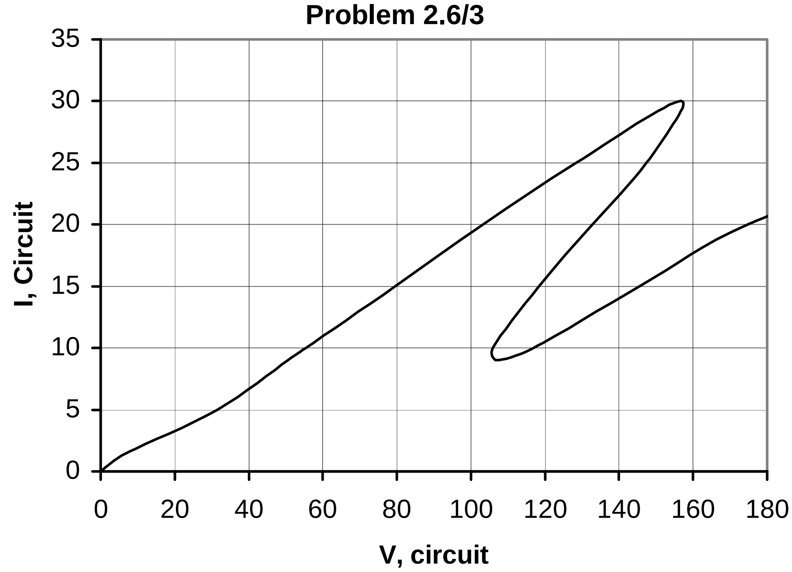
$$P_A = V_A I_A$$

$$P_B = V_B I_B$$

I_B or I_{CIRC}	V_B	V_A	V_{CIRC}
1.5	7.0	5.8	12.8
5.0	12.0	19.5	31.5
9.0	15.6	35.0	50.6
10.0	16.5	38.9	55.4
15.0	21.0	58.4	79.4
20.0	25.5	77.8	103.3
25.0	31.0	97.3	128.3
30.0	40.0	116.7	156.7
25.0	50.0	97.3	147.3
20.0	55.0	77.8	132.8
15.0	60.0	58.4	118.4
10.0	67.0	38.9	105.9
9.0	72.0	35.0	107.0
10.0	78.0	38.9	116.9
15.0	87.0	58.4	145.4
20.0	97.0	77.8	174.8
25.0	124.0	97.3	221.3

Table 2-1 **Calculate $V_{CIRC}\{I_{CIRC}\}$ coordinates, Problem 2.6/3**

Figure 2-7 Circuit electrical behavior, Problem 2.6/3



Problem 2.6/3 cont.

Solution

The circuit in Figure (2-5) has potential operating points at the three intersections in Figure (2-7). The problem statement does not contain sufficient information to uniquely determine the current at 140 volts. At the intersections, the emf, electric current, and power dissipated for Components A and B are listed in Table 2-2.

Component A	Component B
105 volts, 27 amps, 2800 watt	35 volts, 27 amps, 950 watts
86 volts, 22 amps, 1900 watts	54 volts, 22 amps, 1200 watts
54 volts, 14 amps, 760 watts	86 volts, 14 amps, 1200 watts

Table 2-2 Solution of Problem 2.6/3

Problem 2.6/4

Problem statement

What are the values of emf, electric current, and electric power for each component in Figure (2-8)?

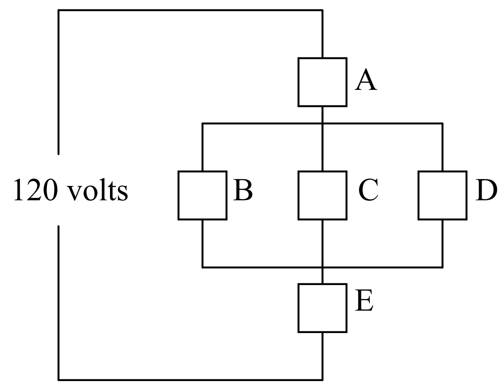


Figure 2-8 **Electric system in Problem 2.6/4**

Given

The electrical behavior of the components in Figure (2-8) is given by Eqs. (2-35) through (2-39).

$$V_A = 4.7\ I_A \quad (2\text{-}35)$$

$$V_B = 3.4\ I_B \quad (2\text{-}36)$$

$$V_C = 5.4\ I_C \quad (2\text{-}37)$$

$$V_D = 4.2\ I_D \quad (2\text{-}38)$$

$$V_E = 2.4\ I_E \quad (2\text{-}39)$$

<u>Problem 2.6/4</u> cont.

Analysis

- Inspect Figure (2-8) and note that:

$$I_B + I_C + I_D = I_{CIRC} \quad (2\text{-}40)$$

$$I_A = I_E = I_{CIRC} \quad (2\text{-}41)$$

$$V_B = V_C = V_D = V_{BCD} \quad (2\text{-}42)$$

$$V_A + V_{BCD} + V_E = 120 \quad (2\text{-}43)$$

- Determine $V_{BCD}\{I_{CIRC}\}$ by combining Eqs. (2-36) to (2-38) and (2-40), and using Eq. (2-42):

$$V_{BCD}/3.4 + V_{BCD}/5.4 + V_{BCD}/4.2 = I_{CIRC} \quad (2\text{-}44)$$

$$\therefore V_{BCD} = 1.394\ I_{CIRC} \quad (2\text{-}45)$$

- Determine $V_{BCD}\{I_{CIRC}\}$ by combining Eqs. (2-35), (2-39), and (2-43), and using Eq. (2-41):

$$V_{BCD} = 120 - 4.7\ I_{CIRC} - 2.4\ I_{CIRC} \quad (2\text{-}46)$$

- Determine V_{BCD} and I_{CIRC} by combining Eqs. (2-45) and (2-46):

$$I_{CIRC} = 14.13 \quad (2\text{-}47a)$$

$$V_{BCD} = 19.7 \quad (2\text{-}47b)$$

- Substitute in Eq. (2-41) to determine I_A and I_E.
- Substitute in Eqs. (2-35) and (2-39) to determine V_A and V_E.
- Substitute in Eq. (2-42) to determine V_B, V_C, and V_D.

$$V_B = V_C = V_D = V_{BCD} = 19.7 \quad (2\text{-}48)$$

- Substitute in Eqs. (2-36) through (2-38) to determine I_B, I_C, and I_D.

Problem 2.6/4 cont.

- Substitute in Eq. (2-4) to determine the power dissipated in each component.

Solution

	volts	**amperes**	**watts**
A	66.4	14.13	938
B	19.7	5.79	114
C	19.7	3.65	72
D	19.7	4.69	92
E	33.9	14.13	479

Table 2-3 **Solution of Problem 2.6/4**

Problem 2.6/5

Problem statement
What are the values of emf, electric current, and electric power for each component in Figure (2-9)?

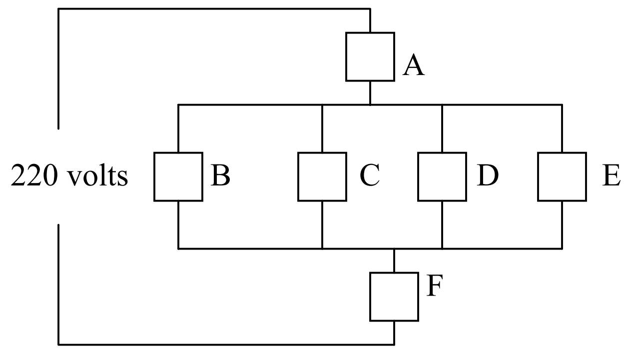


Figure 2-9 **Electric system in Problem 2.6/5**

Given
The electrical behavior of the components in Figure (2-9) is given by Eqs. (2-49) through (2-54)

$$V_A = 1.5\ I_A^{1.3} \tag{2-49}$$

$$V_B = 4.2\ I_B \tag{2-50}$$

$$V_C = 2.6\ I_C^{.70} \tag{2-51}$$

$$V_D = 5.2\ I_D \tag{2-52}$$

$$V_E = 2.1\ I_E^{1.5} \tag{2-53}$$

$$V_F = 1.2\ I_F \tag{2-54}$$

<u>Problem 2.6/5</u> cont.

Analysis

- Inspect Figure (2-9) and note that:

$$(I_B + I_C + I_D + I_E) = I_A = I_F = I_{CIRC} \quad (2\text{-}55)$$

$$V_B = V_C = V_D = V_E = V_{BCDE} \quad (2\text{-}56)$$

$$V_A + V_{BCDE} + V_F = 220 \quad (2\text{-}57)$$

- Determine $V_{BCDE}\{I_{CIRC}\}$ by combining Eqs. (2-50) to (2-53) and (2-55) , and using Eq. (2-56).

$$V_{BCDE}/4.2 + (V_{BCDE}/2.6)^{1.429} + V_{BCDE}/5.2 + (V_{BCDE}/2.1)^{.667} = I_{CIRC} \quad (2\text{-}58)$$

- Determine $V_{BCDE}\{I_{CIRC}\}$ by combining Eqs. (2-49), (2-54), and (2-57), and using Eq. (2-55):

$$V_{BCDE} = 220 - 1.5\, I_{CIRC}^{1.3} - 1.2\, I_{CIRC} \quad (2\text{-}59)$$

- Solve Eqs. (2-58) and (2-59). The result is $V_{BCDE} = 22$, $I_{CIRC} = 35.5$.

- Use the calculated values of V_{BCDE} and I_{CIRC} to sequentially determine:

  - I_A and I_F from Eq. (2-55).

  - V_B, V_C, V_D, and V_E from Eq. (2-56).

  - I_B, I_C, I_D, and I_E from Eqs. (2-50) through (2-53).

  - V_A and V_F from Eqs. (2-49) and (2-54).

- Determine the power dissipated in each component from Eq. (2-4).

Problem 2.6/5 cont.

Solution

For each component in Figure (2-9), the emf, electric current, and power are listed in Table (2-4).

Component	**emf** volts	**electric current** amperes	**power** watts
A	155	35.5	5500
B	22	5.2	115
C	22	21.1	465
D	22	4.2	92
E	22	4.8	105
F	43	35.5	1530

Table 2-4 Solution of Problem 2.6/5

Problem 2.6/6

Problem statement

What are the values of emf and electric current for each component in Figure (2-10)?

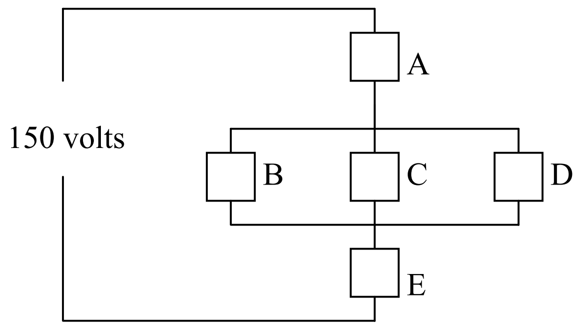


Figure 2-10 Electric system in Problem 2.6/6

Given

The electrical behavior of Components A, B, C, and E is given by Eqs. (2-60) to (2-63). The electrical behavior of Component D is given by Figure (2-11).

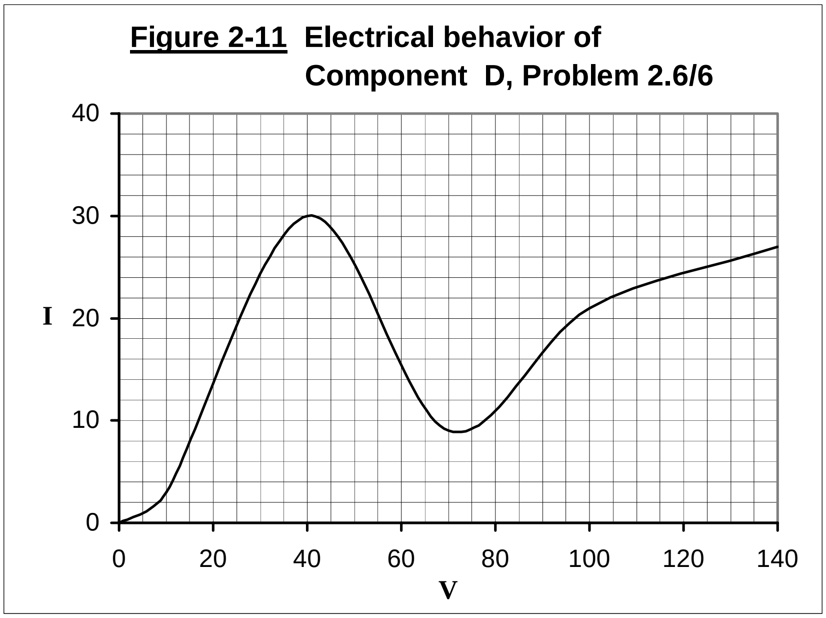
$$V_A = 1.22\ I_A^{1.2} \qquad (2\text{-}60)$$

$$V_B = 12.7\ I_B \qquad (2\text{-}61)$$

$$V_C = 16.3\ I_C \qquad (2\text{-}62)$$

$$V_E = 1.03\ I_E \qquad (2\text{-}63)$$

<u>Problem 2.6/6</u> cont.



Analysis

- Inspect Figure (2-10) and note that:

$$(I_B + I_C + I_D) = I_A = I_E = I_{CIRC} \tag{2-64}$$

$$V_B = V_C = V_D = V_{BCD} \tag{2-65}$$

$$V_A + V_{BCD} + V_E = V_{CIRC} \tag{2-66}$$

- Determine coordinates of $(I_{CIRC})\{V_{CIRC}\}$ in the following way:
  - o List several coordinates of (V_D, I_D) obtained from Figure (2-11).
  - o Calculate $I_B\{V_D\}$ and $I_C\{V_D\}$ from Eqs. (2-61), (2-62), and (2-65).
  - o Add $I_B\{V_D\}$, $I_C\{V_D\}$, and $I_D\{V_D\}$, and obtain $(I_B + I_C + I_D)\{V_D\}$.
  - o Note that $(I_B + I_C + I_D)\{V_D\} = (I_{CIRC})\{V_{BCD}\}$.
  - o Calculate $V_A\{I_{CIRC}\}$ using Eq. (2-60).
  - o Calculate $V_E\{I_{CIRC}\}$ using Eq. (2-63).

<u>Problem 2.6/6</u> cont.

o Calculate V_{CIRC} using Eqs. (2-66) and (2-65).

o The calculations are in Table (2-5).

V_D	I_D	I_B	I_C	I_{CIRC}	V_A	V_E	V_{CIRC}
10	2.6	0.8	0.6	4.0	6.4	4.1	20.6
20	13.7	1.6	1.2	16.5	35.3	17.0	72.3
30	24.2	2.4	1.8	28.4	67.7	29.3	126.9
40	30	3.1	2.5	35.6	88.7	36.7	165.4
50	25.3	3.9	3.1	32.3	79.0	33.3	162.2
60	16.2	4.7	3.7	24.6	57.0	25.3	142.3
70	9	5.5	4.3	18.8	41.3	19.4	130.6
80	11	6.3	4.9	22.2	50.4	22.9	153.2
90	16.7	7.1	5.5	29.3	70.3	30.2	190.5
100	21	7.9	6.1	35.0	87.0	36.1	223.0
110	23	8.7	6.7	38.4	97.2	39.6	246.8
120	24.5	9.4	7.4	41.3	106.1	42.6	268.6
130	25.5	10.2	8.0	43.7	113.5	45.0	288.5
140	27	11.0	8.6	46.6	122.6	48.0	310.6

<u>Table 2-5</u> Calculate (I_{CIRC}, V_{CIRC}) coordinates, Problem 2.6/6

- Plot the $I_{CIRC}\{V_{CIRC}\}$ coordinates from Table (2-5) in Figure (2-12).
- Figure (2-12) indicates three solutions at $V_{CIRC} = 150$: $I_{CIRC} = 22, 28,$ and 33.
- Use the I_{CIRC} solutions to sequentially determine:

o I_A and I_E from Eq. (2-64).

o V_A from Eq. (2-60), V_E from Eq. (2-63).

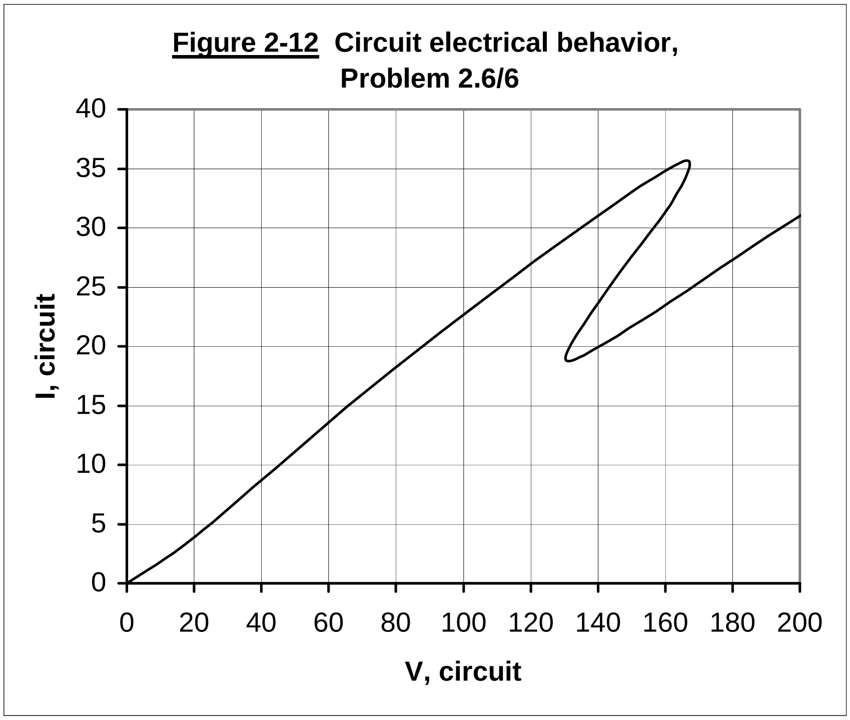
o V_{BCD} from Eq. (2-66).

o V_B, V_C, and V_D from Eq. (2-65).

o I_B and I_C from Eqs. (2-61) and (2-62).

o I_D from Figure (2-11).

<u>Problem 2.6/6</u> cont.



Solution

For each of the three solutions in Figure (2-12), the emf and electric current for the components are given in Table (2-6). The problem statement does not contain sufficient information to determine a unique solution.

V_A	I_A	V_B	I_B	V_C	I_C	V_D	I_D	V_E	I_E
81	33	35	2.8	35	2.1	35	28	34	33
67	28	54	4.3	54	3.3	54	21	29	28
50	22	77	6.1	77	4.7	77	10	23	22

<u>Table 2-6</u> Solution of Problem 2.6/6

2.6 Conclusions

- The problems in this chapter demonstrate how to solve resistive electrical problems using electrical behavior methodology.

- The problems demonstrate that electrical behavior methodology is a simple and direct method for solving proportional problems and nonlinear problems.

- The problems demonstrate analogously that behavior methodology would be useful in other branches of engineering.

Chapter 3

The electrical resistance form of the problems in Chapter 2

3 Introduction

In Chapter 2, electrical problems are stated in behavior form, and are solved using electrical behavior methodology. In this chapter, the problems in Chapter 2 are stated in resistance form, and are to be solved by the reader.

Corresponding problems, figures, and equations in this chapter have the same identifying numbers used in Chapter 2, except that "R" is added to the identifying numbers (to denote resistance form). For example, Problem (2.5/3R) in this chapter is the resistance form of Problem (2.5/3) in Chapter 2. Eq. (2-23R) in this chapter is the resistance form of Eq. (2-23) in Chapter 2.

The reader is encouraged to solve the problems (particularly the non-linear problems) using resistance methodology. By comparing her/his resistance solutions with the behavior solutions presented in Chapter 2, the reader will gain a first hand appreciation of the simplicity that results from using electrical behavior methodology rather than electrical resistance methodology.

3.1 The definition of electrical "resistance"

Recall the quote from Maxwell (1873) cited above:

> *(Ohm's law states that) the resistance of a conductor . . . is defined to be the ratio of the electromotive force to the strength of the current which it produces.*

Also recall the definition in encyclopedia Britannica (1999-2000):

> *Precisely, $R = V/I$.*

In other words, *by definition*, electrical "resistance" is the ratio V/I. This ratio is assigned the symbol R and the dimension "ohms".

3.2 The widely accepted view of *V/I* (symbol *R*)

In the 19th century, it was felt that *all* conductors of electricity exhibited proportional behavior in accordance with Ohm's law. The above quote from Maxwell (1873) continues:

> *The resistance of a conductor may be measured to within one ten thousandth . . . and so many conductors have been tested that our assurance of the truth of Ohm's law* (ie that *V* is globally proportional to *I*) *is now very high.*

Since all conductors exhibited proportional behavior, it was not germane to ask

> Should *V/I* (symbol *R*) be used to solve only proportional problems? Or should *V/I* also be used to solve nonlinear problems?

Today, many important electrical devices exhibit nonlinear behavior. It long ago became germane to question whether *V/I* should be used to solve nonlinear problems as well as proportional problems.

The widely accepted conventional engineering view is that *V/I* (symbol *R*) should be used to solve proportional problems, but should not be used to solve nonlinear problems. Nonlinear problems should be solved using methodology that is not based on *V/I* (symbol *R*).

Note that in the widely accepted conventional view, *two* methodologies are required in order to solve both proportional and nonlinear problems.

Also note that behavior methodology *alone* is required in order to solve both proportional and nonlinear problems.

3.3 An alternative view of *V/I* (symbol *R*)

It is not universally accepted that *V/I* (symbol *R*) should be used to solve only problems that concern proportional behavior. For example, an alternative view is expressed by Halliday and Resnick (1978):

- Ohm's law is *not* the expression $V = IR$. This expression merely defines *R* to be a symbol for the ratio *V/I*.
- Ohm's law is the observation that *V/I* (symbol *R*) is independent of *I* for a certain class of conductors.

- *V/I* (symbol *R*) should be used whether or not *V/I* is independent of *I*—ie *V/I* should be used to solve problems that concern *all* forms of electrical behavior—proportional, linear, and nonlinear.

Based on this alternative view, *V/I* (symbol *R*) can and should be used to solve all the resistive electrical problems in this book.

3.4 A preview of the problems

Problems 2.5/1R, 2.6/1R, and 2.6/4R concern proportional circuits—ie they concern circuits that include components that exhibit proportional relationships between *V* and *I*. This behavior is so simple that both the behavior analyses and the resistance analyses are simple and direct. However, the reader should note the following:

- When resistance methodology is used, problems are stated and solutions are presented in terms of *V* and *I*. But relationships are described and analyses are performed in terms of *V/I (*symbol *R)*.

- When behavior methodology is used, problems are stated, relationships are described, analyses are performed, and solutions are presented in terms of V and I.

Note that behavior methodology is more logical than resistance methodology because there is no good reason to use *V* and *I* for problem statements and solutions, and *V/I* for descriptions and analyses.

Problem 2.5/2R concerns a component that exhibits moderately non-linear resistance, and Problem 2.6/2R concerns a series connected circuit in which one component exhibits moderately nonlinear resistance. These problems are sufficiently simple to be solved in a direct manner using resistance methodology.

Problem 2.6/5R concerns a series-parallel connected circuit that contains a moderately nonlinear component. Problems 2.5/3R, 2.6/3R, and 2.6/6R concern circuits that include a component that exhibits highly nonlinear resistance. Problem 2.5/3R concerns a single component, Problem 2.6/3R concerns a series connected circuit, and Problem 2.6/6R concerns a series-parallel connected circuit. These problems must be solved in an indirect manner if resistance methodology is used.

3.5 The resistance form of the problems in Chapter 2

Problem 2.5/1R

Problem statement
In Figure (2-1R), what power supply emf would cause a current of 7.2 amperes? What power would be dissipated in Component A?

Given
The electrical resistance of Component A is described by Eq. (2-5R).

$$R_A = 5.6 \text{ ohms} \tag{2-5R}$$

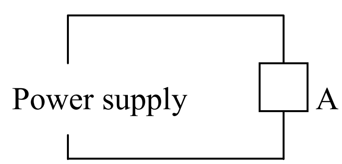


Figure 2-1R Electric system, Problems 2.5/1R to 2.5/3R

Analysis and Solution
(To be determined by the reader.)

Problem 2.5/2R

Problem statement

In Figure (2-1R), what current would be caused by a power supply emf of 75 volts? What power would be dissipated in Component A?

Given

The electrical resistance of Component A is described by Eq. (2-8R). (Eq. (2-8R) is inhomogeneous. The *I* dimension is amps.)

$$R_A = 4.7\, I_A^{0.4} \text{ ohms} \tag{2-8R}$$

Analysis and Solution

(To be determined by the reader.)

<u>Problem 2.5/3R</u>

Problem statement

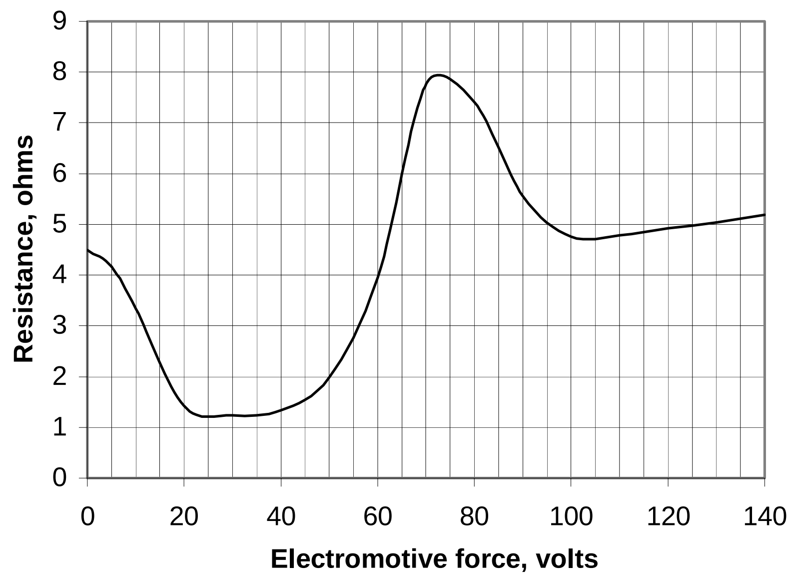
In Figure (2-1R), what power supply emf would cause a current of 20 amps? What power would be dissipated in Component A?

Given

The electrical resistance of Component A is described in Figure (2-2R). Note that the chart cannot be read in a direct manner if *I* is given because *I* is not shown explicitly. (*I* is implicit in *R*, the symbol for *V/I.)*

To solve Problem 2.5/3R, the chart must be read in an indirect manner, for example by estimating the value of *V*, and using the chart and the known value of current to iteratively improve the estimate.

Figure 2-2R Resistance of Component A, Problem 2.5/3R



Analysis and Solution

(To be determined by the reader.)

Problem 2.6/1R

Problem statement

What are the values of emf, electric current, and electric power for each component in Figure (2-3R)?.

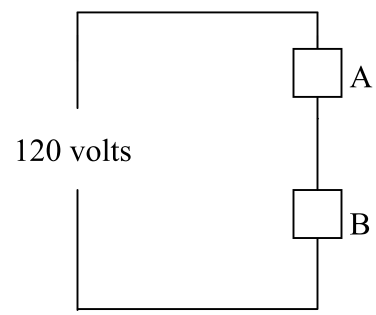


Figure 2-3R **Electric system, Problem 2.6/1R**

Given

The electrical resistance of Components A and B is described by Eqs. (2-13R) and (2-14R).

$$R_A = 17 \text{ ohms} \tag{2-13R}$$

$$R_B = 9.4 \text{ ohms} \tag{2-14R}$$

Analysis and Solution

(To be determined by the reader.)

Problem 2.6/2R

Problem statement
What are the values of emf, electric current, and electric power for each component in Figure (2-4R)?

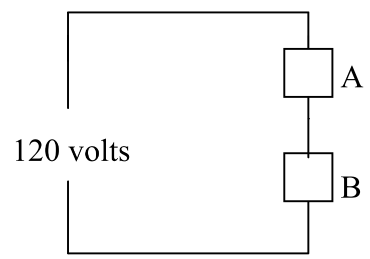


Figure 2-4R **Electric system, Problem 2.6/2R**

Given
The electrical resistance of Components A and B is described by Eqs. (2-22R) and (2-23R). (Eq. (2-23R) is inhomogeneous. The *I* dimension is amps.)

$$R_A = 3.6 \text{ ohms} \tag{2-22R}$$

$$R_B = 4.8\, I_B^{\,0.5} \text{ ohms} \tag{2-23R}$$

Analysis and Solution
(To be determined by the reader.)

Problem 2.6/3R

Problem statement

What are the values of emf, electric current, and electric power for each component in Figure (2-5R)?

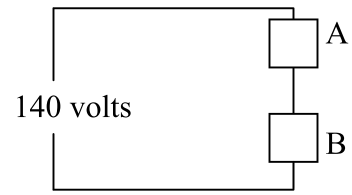


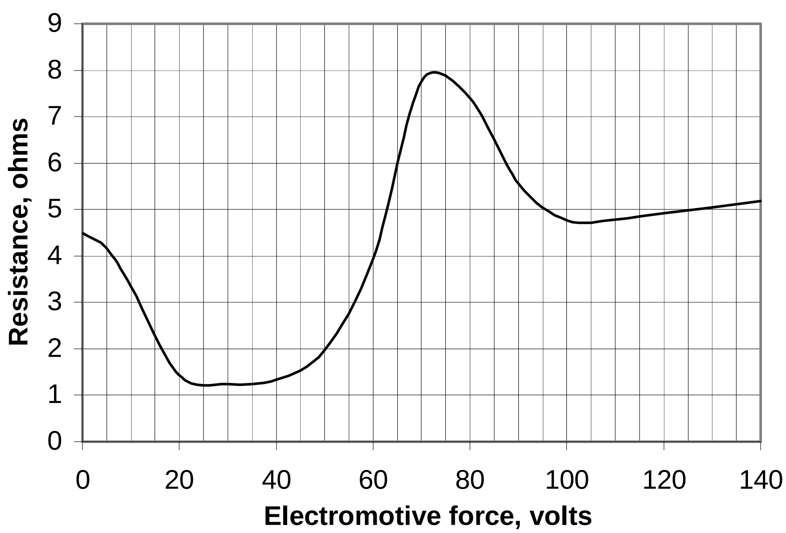
Figure 2-5R Electric system in Problem 2.6/3R

Given

The electrical resistance of Component A is given by Eq. (2-31R). The electrical resistance of Component B is given by Figure (2-6R).

$R_A = 3.89$ ohms (2-31R)

Figure 2-6R Resistance of Component B, Problem 2.6/3R



Analysis and solution

(To be determined by the reader.)

Problem 2.6/4R

Problem statement

What are the values of emf, electric current, and electric power for each component in Figure(2-8R)?

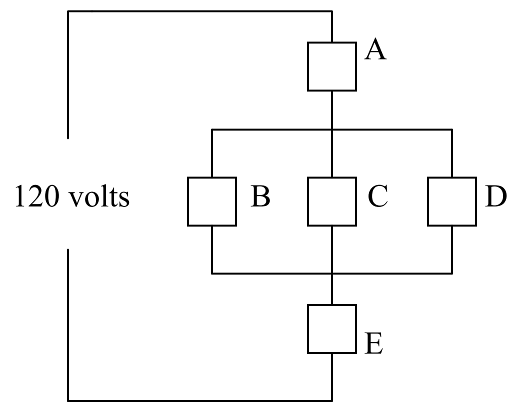


Figure 2-8R Electric system, Problem 2.6/4R

Given

The electrical resistance of the components in Figure (2-8R) is described by Eqs. (2-35R) through (2-39R).

$$R_A = 4.7 \text{ ohms} \tag{2-35R}$$

$$R_B = 3.4 \text{ ohms} \tag{2-36R}$$

$$R_C = 5.4 \text{ ohms} \tag{2-37R}$$

$$R_D = 4.2 \text{ ohms} \tag{2-38R}$$

$$R_E = 2.4 \text{ ohms} \tag{2-39R}$$

Analysis and Solution

(To be determined by the reader.)

Problem 2.6/5R

Problem statement
What are the values of emf, electric current, and electric power for each component in Figure (2-9R)?

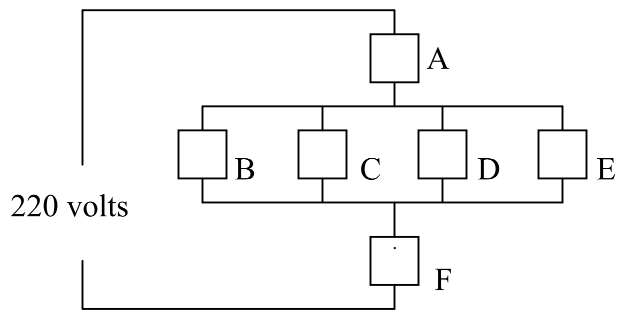


Figure 2-9R Electric system, Problem 2.6/5R

Given
The electrical resistance of the components in Figure (2-8R) is described by Eqs. (2-49R) through (2-54R). (Eqs. (2-49R), (2-51R), and (2-53R) are dimensional equations in which the *I* dimension is amps.)

$$R_A = 1.5\, I_A^{\,0.3} \text{ ohms} \tag{2-49R}$$

$$R_B = 4.2 \text{ ohms} \tag{2-50R}$$

$$R_C = 2.6\, I_C^{\,-.30} \text{ ohms} \tag{2-51R}$$

$$R_D = 5.2 \text{ ohms} \tag{2-52R}$$

$$R_E = 2.1\, I_E^{\,0.5} \text{ ohms} \tag{2-53R}$$

$$R_F = 1.2 \text{ ohms} \tag{2-54R}$$

Analysis and Solution
(To be determined by the reader.)

Problem 2.6/6R

Problem statement

What are the values of emf, electric current, and electric power for each component in Figure (2-10R)?

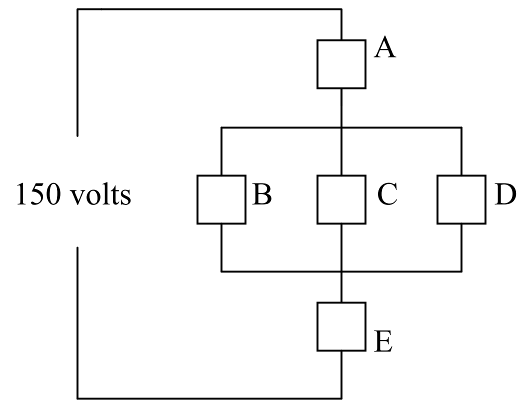


Figure 2-10R **Electric system, Problem 2.6/6R**

Given

The electrical resistance of Component D is described in Figure (2-11R). The electrical resistance of Components A, B, C, and E are described in Eqs. (2-60R) through (2-63R). (Eq. (2-60R) is a dimensional equation in which the *I* dimension is amps.)

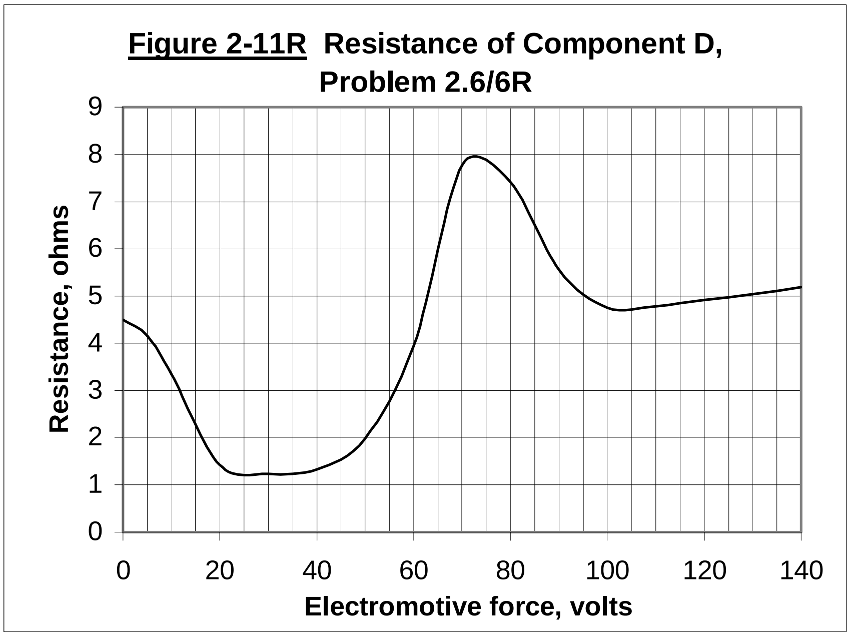
$$R_A = 1.22\ I_A^{0.2} \text{ ohms} \quad (2\text{-}60R)$$

$$R_B = 12.7 \text{ ohms} \quad (2\text{-}61R)$$

$$R_C = 16.3 \text{ ohms} \quad (2\text{-}62R)$$

$$R_E = 1.03 \text{ ohms} \quad (2\text{-}63R)$$

<u>Problem 2.6/6R</u> cont.



Analysis and Solution

(To be determined by the reader.)

3.6 Conclusions

- Proportional problems can be solved in a direct manner using behavior methodology or resistance methodology.

- Nonlinear electrical problems that must be solved in an indirect manner using resistance methodology can be solved in a direct and much simpler manner using behavior methodology.

Chapter 4

Why electrical behavior V{I} should replace electrical resistance *V/I*

4 Introduction

This chapter addresses the question

> Should electrical "behavior" replace electrical "resistance"?

The question is answered in two ways:

- In a general way by appraising and comparing behavior and resistance concepts and methodologies.

- In a specific way by comparing the behavior analyses in Chapter 2 with the resistance analyses of the same problems in Chapter 3.

The answers strongly support the conclusion that electrical "behavior" V{I} should replace electrical "resistance" *V/I*.

4.1 The de facto view of electrical resistance

For almost 200 years, electrical resistance has been used to describe, analyze, and predict electrical phenomena. Resistance and resistance methodology are so fundamental and so simple that they are generally studied in high school physics.

The end result of this long history and youthful exposure is that electrical resistance has come to be viewed as a fundamental parameter of Nature—a parameter as real as electromotive force or temperature—a parameter whose existence *cannot* be denied.

Based on the de facto view of electrical resistance, it is preposterous to question its value.

4.2 The true nature of "resistance"

Recall Maxwell's (1873) observation that

> *. . . the resistance of a conductor . . . is defined to be the ratio of the electromotive force to the strength of the current which it produces.*

Also recall the definition in encyclopedia Britannica (1999-2000):

> *Precisely, R = V/I.*

The above definitions state that

- "Resistance" is a *not* an electrical parameter. "Resistance" is a *ratio* of electrical parameters—the ratio *V/I*.

- *"R"* is *not* a symbol for an electrical parameter. *"R"* is a symbol for a *ratio* of electrical parameters—the ratio *V/I*.

- The dimension "ohms" is *not* the dimension of an electrical parameter. "Ohms" is the dimension of a *ratio* of electrical parameters in specific dimensions—the ratio *V/I* in which *V* is in volts, and *I* is in amperes

In short, "resistance" is *not* a parameter found in Nature. It is a *contrived* parameter created by combining *V* and *I* in the ratio *V/I*. This ratio is assigned the name resistance, the symbol *R*, and the dimension ohms.

Because electrical resistance is merely a contrived parameter created by combining primary parameters, there is good reason to appraise its value, and to ask whether it should be retained or abandoned.

4.3 The changing view of Ohm's law and *V/I* (symbol *R*)

When it was originally agreed that the ratio *V/I* would be assigned the name "resistance" and the symbol *R*, the data obtained from *all* conductors indicated that *V/I* was independent of current. For example, the above quote from Maxwell (1873) is taken from the following:

> *(Ohm's law states that) the resistance of a conductor . . . is defined to be the ratio of the electromotive force to the strength of the current which it produces. . . . .*
>
> *In the first place, then, the resistance of the conductor is independent of the strength of the current flowing through it . . .*
>
> *The resistance of a conductor may be measured to within one ten thousandth . . . and so many conductors have been tested that our assurance of the truth of Ohm's law is now very high.*

In other words:

- Ohm's law states that electrical resistance is *V/I* (symbol *R*), and that *V/I* is independent of *I*.
- Many conductors were tested to determine whether *V/I* is in fact independent of *I* in accordance with Ohm's law.
- Without exception, it was found that *V/I* is independent of *I*.
- Ohm's law is accepted as a true law because tests of conductors indicate that *V/I* is *globally* independent of I.

Decades after the above quote, the development of nonlinear electrical devices made it necessary to recognize that *V/I* (symbol *R*) is oftentimes strongly dependent on *I*. Since this is at odds with the original view that *V/I* is globally independent of *I*, it became necessary to either reinterpret Ohm's law and *V/I*, or abandon them.

In conventional engineering, Ohm's law and *V/I* (symbol *R*) have been reinterpreted in two ways. Recall from Chapter 3 that:

- The widely held view is that Ohm's law states that *V/I* (symbol *R*) is independent of *I*. *V/I* should be used to solve only proportional problems. Nonlinear problems should be solved using methodology *not* based on *V/I* (symbol *R*).
- The alternate view is that Ohm's law states that *V/I* (symbol *R*) is independent of *I* for a certain type of conductor. *V/I* should be used to solve problems whether or not *V/I* (symbol *R*) is independent of I.

In the new engineering, Ohm's law and *V/I* (symbol *R*) and resistance methodology are abandoned. They are replaced by behavior equations and behavior methodology.

4.4 The true nature of electrical "behavior"

Recall from Chapter 1 that

> Resistive electrical "behavior" is the relationship between emf and the strength of the electric current it produces. In short, it is V = *f*{I} or I = *f*{V}.

4.5 The identical relationship of resistance methodology and behavior methodology

Resistance methodology and behavior methodology are *identical.* They differ *only* in form.

- In resistance methodology, the primary parameters appear in the forms *V*, *I*, and *V/I* (symbol *R*).

- In behavior methodology, the primary parameters appear only in the forms V and I.

In order to demonstrate that behavior methodology is identical to resistance methodology, it is sufficient to show that resistance equations can be transformed to behavior equations, and conversely. The transformation is accomplished by substituting (*V/I*) for *R*, then separating *V* and *I*, and conversely.

For example, Eq. (4-1) is used in resistance methodology:

$$R_{PARALLEL} = (\Sigma R_i^{-1})^{-1} \qquad (4\text{-}1)$$

> *The electrical resistance of parallel components is equal to the reciprocal of the sum of the reciprocals of the resistances of the parallel components.*

The transformation of Eq. (4-1) from resistance form to behavior form is accomplished in the following manner:

- Substitute (V/I) for *R* in Eq. (4-1).

$$(V/I)_{PARALLEL} = (\Sigma\ (V_i/I_i)^{-1})^{-1} \qquad (4\text{-}2)$$

- Separate V and I by noting that, because the components are in parallel, all the V's are equal, and therefore Eq. (4-2) can be written

$$(V/I)_{PARALLEL} = V/\Sigma\, I_i \qquad (4\text{-}3)$$

$$\therefore I_{PARALLEL} = \Sigma\, I_i \qquad (4\text{-}4)$$

The electric currents through parallel components are additive.

Eq. (4-4) is the behavior form of Eq. (4-1). In behavior methodology, it altogether replaces Eq. (4-1).

The transformation of Eq. (4-1) to Eq. (4-4) demonstrates that resistance methodology and behavior methodology differ only in form.

4.6 Choosing between resistance methodology and behavior methodology

The choice between resistance methodology and behavior methodology can be based on several factors:

- Simplicity of problem solutions.
- Generality.
- Logical basis.
- Ease of learning.

4.6.1 Simplicity of problem solutions

The problems in Chapters 2 and 3 demonstrate that:

- Proportional electrical problems can be solved in a simple and direct manner using resistance methodology or behavior methodology.
- Nonlinear electrical problems that must be solved in an indirect manner using resistance methodology can be solved in a direct and much simpler manner using behavior methodology.

Therefore, based on the simplicity of problem solutions, behavior methodology is preferable to resistance methodology.

4.6.2 Generality

In the widely accepted view, resistance methodology should be used to solve *only* proportional problems. Nonlinear problems should be solved using methodology not based on *V/I* (symbol *R*).

Behavior methodology is used to solve *both* proportional and nonlinear problems.

Therefore, based on generality, behavior methodology is preferable to resistance methodology.

4.6.3 Logical bases

The logical bases of resistance methodology and behavior methodology are revealed by noting that:

- In resistance methodology, *V*, *I*, *and V/I* (symbol *R*) are used to solve problems that concern *V* and *I*. Note that *V/I* is a variable if *V* is not proportional to *I*, in which case *three* variables are used to solve problems that concern *two* variables. It is not logical to use three variables to solve problems that concern two variables.

- In behavior methodology, V and I are used to solve problems that concern V and I. It is logical to use two variables to solve problems that concern two variables.

Therefore, using logical bases as the criterion, behavior methodology is preferable to resistance methodology.

4.6.4 Ease of learning

Table (4-1) is a behavior/resistance dictionary. With regard to ease of learning, notice that:

- If all reference to resistance is deleted from resistance methodology, the result is behavior methodology. Thus behavior methodology is easier to learn because there is less to learn.

- In the widely accepted conventional view, the resistance methodology described in Table (4-1) is used to solve proportional problems; different methodology is used to solve nonlinear problems. The behavior methodology described in Table (4-1) is used to solve *both* proportional problems and nonlinear problems. Thus behavior methodology is easier to learn because it obviates the need to learn a second methodology.

- The meaning and validity of the equations and expressions regarding parallel components are much more readily apparent in behavior methodology.

Therefore, based on ease of learning, behavior methodology is preferable to resistance methodology.

4.7 Maxwell on the value of electrical resistance in 1873

The quote from Maxwell (1873) cited above is an excerpt from the following:

> *(Ohm's law states that) the (electrical) resistance of a conductor . . . is defined to be the ratio of the electromotive force to the strength of the current which it produces. The introduction of this term would have been of* ***no scientific value*** *unless Ohm had shown, as he did experimentally, that . . . it has a definite value which is altered only when the nature of the conductor is altered.*
>
> *In the first place, then, the resistance of the conductor is independent of the strength of the current flowing through it . . .*
>
> *The resistance of a conductor may be measured to within one ten thousandth . . . and so many conductors have been tested that our assurance of the truth of Ohm's law is now very high.*

In other words, electrical resistance was a valuable concept in 1873 because many conductors had been tested, and they *all* obeyed Ohm's law—they *all* indicated that V/I is "independent of the strength of the current flowing through it".

Maxwell also states that, if some of the conductors tested had indicated that V/I was *dependent* on I, then Ohm's law would *not* be true, and electrical resistance would have *no scientific value*.

Table 4-1 Behavior/Resistance Dictionary

Behavior methodology	Resistance methodology
If components are connected in series, emf's are additive.	If components are connected in series, emf's are additive. If components are connected in series, resistances are additive.
$V_{SERIES\ TOTAL} = \Sigma\ V_{COMP}$	$V_{SERIES\ TOTAL} = \Sigma V_{COMP}$ $R_{SERIES\ TOTAL} = \Sigma R_{COMP}$
If components are connected in series, currents are equal.	If components are connected in series, currents are equal.
If components are connected in parallel, currents are additive.	If components are connected in parallel, currents are additive. If components are connected in parallel, the total resistance is the reciprocal of the sum of the reciprocals of the resistances.
$I_{PARALLEL} = \Sigma\ I_i$	$I_{PARALLEL} = \Sigma I_i$ $R_{PARALLEL} = (\Sigma R_i^{-1})^{-1}$
If components are connected in parallel, emf's are equal.	If components are connected in parallel, emf's are equal.
$P = VI$	$P = VI$ $P = I^2 R$ $P = V^2/R$

4.8 The value of *V/I* today

For approximately 100 years, many devices have not "obeyed" Ohm's law—their resistance *V/I is strongly dependent on I*. There can be little doubt that, were Maxwell alive today, his view would be:

- Ohm's law is *not* a true law because it does not apply globally.
- The ratio *V/I* (symbol *R*) has *"no scientific value"* because it does *not* generally have *"a definite value which is altered only when the nature of the conductor is altered"*.
- Ohm's law and the ratio *V/I* and the word "resistance" and the symbol *R* and resistance methodology should all be *abandoned*.

4.9 Summary

Electrical behavior methodology has the following advantages relative to resistance methodology:

- It greatly simplifies the solution of nonlinear problems in general because it allows the primary parameters V and I to be separated rather than combined in the ratio *V/I* (symbol *R*).
- It is more generally applicable because it can be used to solve proportional problems *and* nonlinear problems. In the widely accepted conventional view, resistance methodology is used to solve proportional problems, and different methodology is used to solve nonlinear problems.
- It has a more logical basis because problems that concern two variables (V and I) are solved using *two* variables (V and I) rather than *three* variables (*V* and *I* and *V/I* (symbol *R*)).
- It is easier to learn because
    - It *alone* must be learned in order to solve proportional problems *and* nonlinear problems. In the widely accepted conventional view of resistance, *two* methodologies must be learned.
    - There is less to learn because all of the analytical expressions used in behavior methodology are also used in resistance methodology.

- o It is analogous to methodology learned in undergraduate mathematics in that problems are solved with the variables separated.

Electrical resistance methodology has only one advantage relative to behavior methodology. It is currently used globally.

4.10 Conclusions

- Electrical science should abandon Ohm's law, *V/I*, the symbol *R*, the word "resistance", and resistance methodology.

- Electrical behavior methodology should be used to describe, analyze, and predict all forms of resistive electrical behavior.

- By analogy, *all* parameters created by combining primary parameters should be abandoned in favor of behavior methodology.

Chapter 5

Stability of resistive electrical systems

5 Introduction

Instability in resistive electrical systems is a practical problem only if a component exhibits such highly nonlinear behavior that dI/dV is negative over some part of the system operating range.

In this chapter, the stability and performance of resistive electrical systems are analyzed using behavior methodology. The analyses can also be performed using resistance methodology, but the extreme nonlinearity involved causes stability analyses based on resistance methodology to be so difficult there is little point in attempting them.

The problems in this chapter illustrate that behavior methodology deals simply and effectively with resistive electrical systems, even if they contain components that exhibit the extremely nonlinear behavior described in Figures (5-3) and (5-7).

5.1 The stability question

The stability analyses in this chapter answer the question:

> If a system is initially at a potential operating point, will the system resist a *very small* perturbation, and return to the potential operating point?

If the answer is "no", the system is "unstable" at the potential operating point—ie it *will not* operate in a steady-state manner at that point. However, it may be quite stable at other potential operating points.

If the answer is "yes", the system is *conditionally* "stable" at the potential operating point—ie it will operate in a steady-state manner at that point *provided* all perturbations are small. The system is only *conditionally* stable at the potential operating point because, even though it is stable with respect to *small* perturbations, it may be unstable with respect to *large* perturbations.

5.2 The effect of instability

If a system is initially at an unstable operating point and is left alone, the system will tend to leave the unstable point. One of the following will result:

- Hysteresis.
- Undamped oscillation.

The electrical behavior of the components determines whether instability results in hysteresis or undamped oscillation.

5.3 Uncoupling the system in order to analyze stability

In system analysis, it is often convenient to:

- Uncouple the system—ie divide it into subsystems.
- Analytically determine the behavior of each subsystem.
- Analytically determine the system performance that would result from coupling the subsystems.

The above method is used here to analyze the stability of electrical systems. The systems analyzed contain a power supply and a circuit of several components, one of which exhibits highly nonlinear electrical behavior that includes a region in which (dI/dV) is negative. The method includes the following steps:

- Uncouple the system to obtain two subsystems. One subsystem contains the highly nonlinear component and all components connected in parallel with it. The other subsystem contains the rest of the system including the power supply.
- Since the emf rises in the subsystem that includes the power supply, the subscript "RISE" is used to refer to this subsystem.
- Since the emf falls in the subsystem that includes the highly nonlinear component, the subscript "FALL" is used to refer to this subsystem.

- Determine $I_{FALL}\{V_{FALL}\}$.
- Determine $I_{RISE}\{V_{RISE}\}$.
- Plot $I_{FALL}\{V_{FALL}\}$ and $I_{RISE}\{V_{RISE}\}$ together on the same graph.
- Note that intersections of $I_{FALL}\{V_{FALL}\}$ and $I_{RISE}\{V_{RISE}\}$ are potential operating points.
- Use Criterion (5-1) to appraise the stability of the system at potential operating points.

5.4 The criterion for electrical system instability

Criterion (5-1) is the criterion for electrical system instability:

$$(dI/dV)_{RISE} \geq_U (dI/dV)_{FALL} \qquad (5\text{-}1)$$

The criterion states:

> If a subsystem in which the emf rises is coupled to a subsystem in which the emf falls, the resultant system will be *unstable* at a potential operating point if $(dI/dV)_{RISE}$ is greater than or equal to $(dI/dV)_{FALL}$. (The $\geq_U$ symbolism indicates "unstable if satisfied".)

The criterion describes stability with regard to *very small* perturbations. Therefore:

- If the criterion is satisfied at a potential operating point, the system is unstable at that potential operating point with regard to very small perturbations.
- If the criterion is *not* satisfied at a potential operating point, the system is stable at that potential operating point with respect to very small perturbations. However, it may or may not be stable with respect to large perturbations.

In this chapter, a system is described as "stable" at a potential operating point if Criterion (5-1) is unsatisfied. However, it must be recognized that "stable" is used as a shorthand expression for "stable with regard to very small perturbations".

The system design objective is generally "stable with respect to perturbations inherent in the system". Fortunately, background perturbations in real systems are generally quite small. Thus there is usually little practical difference between "stable with respect to small perturbations", and "stable with respect to perturbations inherent in the system".

5.5 Verifying Criterion (5-1)

Criterion (5-1) can be verified by showing that, if a circuit is initially at a potential operating point at which the criterion is satisfied, a small perturbation will tend to increase with time.

Figure (5-1) describes the electrical behavior of two subsystems, $I_{FALL}\{V_{FALL}\}$ and $I_{RISE}\{V_{RISE}\}$, in the vicinity of an intersection. If the two subsystems are connected in a circuit, the stability of the circuit at the intersection can be appraised in the following manner:

- Assume that the system described in Figure (5-1) is initially operating at the intersection.

- Suddenly the system experiences a very small, positive perturbation in V.

- The positive perturbation causes I_{RISE} to be greater than I_{FALL}.

- Because I_{RISE} is greater than I_{FALL}, V *increases* with time.

- An increasing V indicates that the positive perturbation is growing, and that the system is not returning to the potential operating point. Therefore the intersection in Figure (5-1) is an unstable operating point.

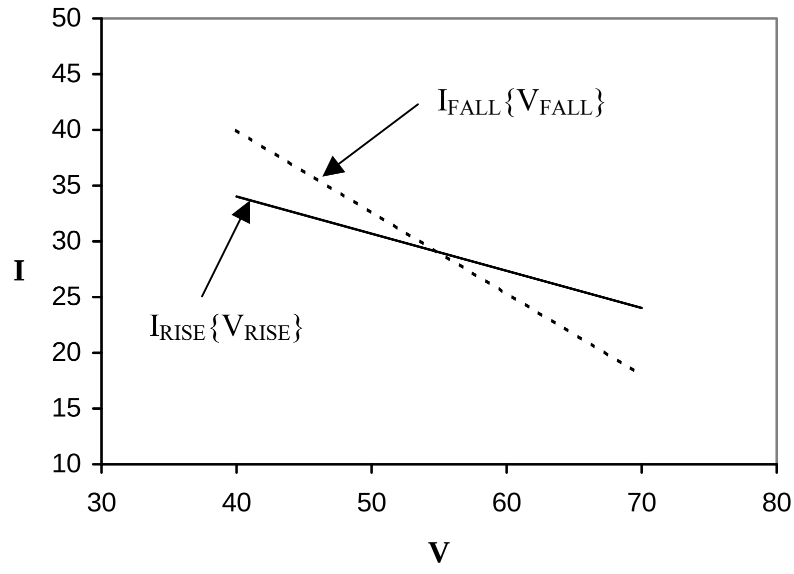


Figure 5-1 Potential operating point

- To determine whether Criterion (5-1) also indicates instability, note that the slope of $I_{RISE}\{V_{RISE}\}$ is greater than the slope of $I_{FALL}\{V_{FALL}\}$. Since this satisfies Criterion (5-1), the criterion indicates instability. (Note that, since both slopes are negative, the *greater* slope is *less* steep.)

- Since the above analysis and Criterion (5-1) are in agreement, the analysis validates Criterion (5-1).

- If $I_{FALL}\{V_{FALL}\}$ and $I_{RISE}\{V_{RISE}\}$ were interchanged, a positive perturbation in V would cause V to decrease, Criterion (5-1) would not be satisfied, and the system would be stable at the intersection.

5.6 Stability analysis of an electrical system—Problem 5.6

Problem 5.6 demonstrates:

- How to analyze an electrical system for instability.
- How to determine the effect of electrical system instability on system performance.

Problem statement

Describe the performance of the system in Figure (5-2) over its operating range of 0 to 400 volts. In other words, determine $I_{PS}\{V_{PS}\}$ for $V_{PS} = 0$ to 400. (Note that subscript PS refers to power supply.)

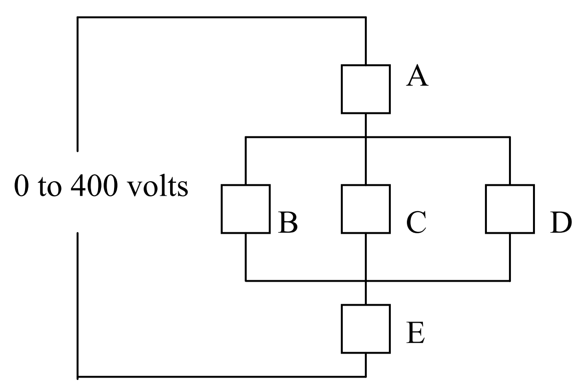


Figure 5-2 Electric Circuit in Problem 5.6

Given

The electrical behavior of Components A, B, C, and E is given by Eqs. (5-2) to (5-5). The behavior of Component D is given by Figure (5-3).

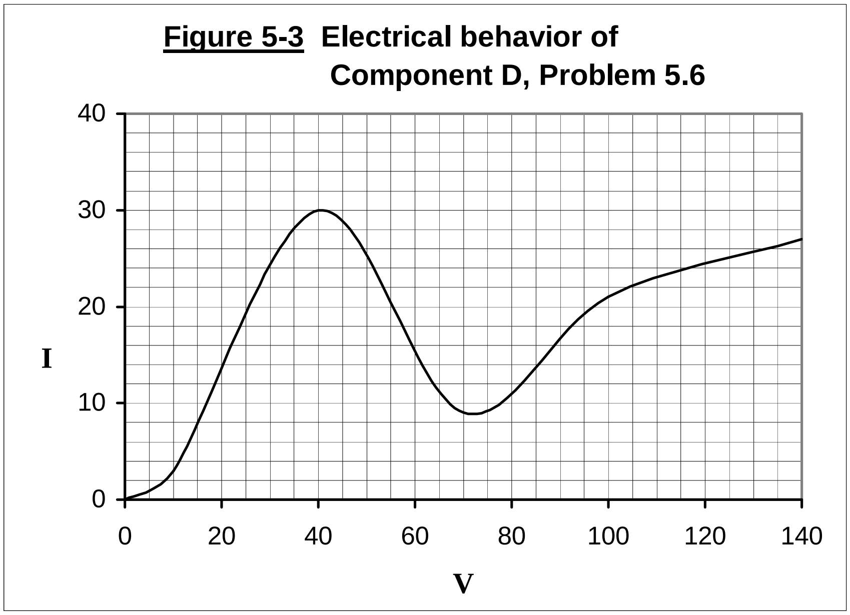
$$V_A = 1.8\ I_A \quad (5\text{-}2)$$

$$V_B = 12.7\ I_B \quad (5\text{-}3)$$

$$V_C = 16.3\ I_C \quad (5\text{-}4)$$

$$V_E = 4.5\ I_E \quad (5\text{-}5)$$

Problem 5.6 cont.



Analysis

- Uncouple the system to obtain a subsystem that contains Components B, C, and D. Use FALL to refer to the BCD subsystem, RISE to refer to the rest of the system including the power supply.

- Inspect Figure (5-2) and note that:

$$I_B + I_C + I_D = I_A = I_E = I_{PS} = I_{RISE} = I_{FALL} \qquad (5\text{-}6)$$

$$V_{FALL} = V_B = V_C = V_D \qquad (5\text{-}7)$$

$$V_{RISE} = V_{PS} - V_A - V_E \qquad (5\text{-}8)$$

- Write an equation for $I_{RISE}\{V_{RISE}\}$ by substituting Eqs. (5-2) and (5-5) in Eq. (5-8), and using Eq. (5-6).

$$V_{RISE} = V_{PS} - 1.8I_{RISE} - 4.5I_{RISE} \qquad (5\text{-}9a)$$

$$\therefore I_{RISE} = 0.1587(V_{PS} - V_{RISE}) \qquad (5\text{-}9b)$$

Problem 5.6 cont.

- Determine coordinates of $I_{FALL}\{V_{FALL}\}$ in the following way:
    - List several coordinates of (V_D, I_D) obtained from Figure (5-3).
    - At each coordinate, calculate $I_B\{V_D\}$ and $I_C\{V_D\}$ from Eqs. (5-3), (5-4), and (5-7).
    - At each coordinate, add $I_B\{V_D\}$, $I_C\{V_D\}$, and $I_D\{V_D\}$, and obtain $(I_B + I_C + I_D)\{V_D\}$.
    - Note that $(I_B + I_C + I_D)\{V_D\} = I_{FALL}\{V_{FALL}\}$.
    - The $I_{FALL}\{V_{FALL}\}$ calculations and results are listed in Table (5-1).

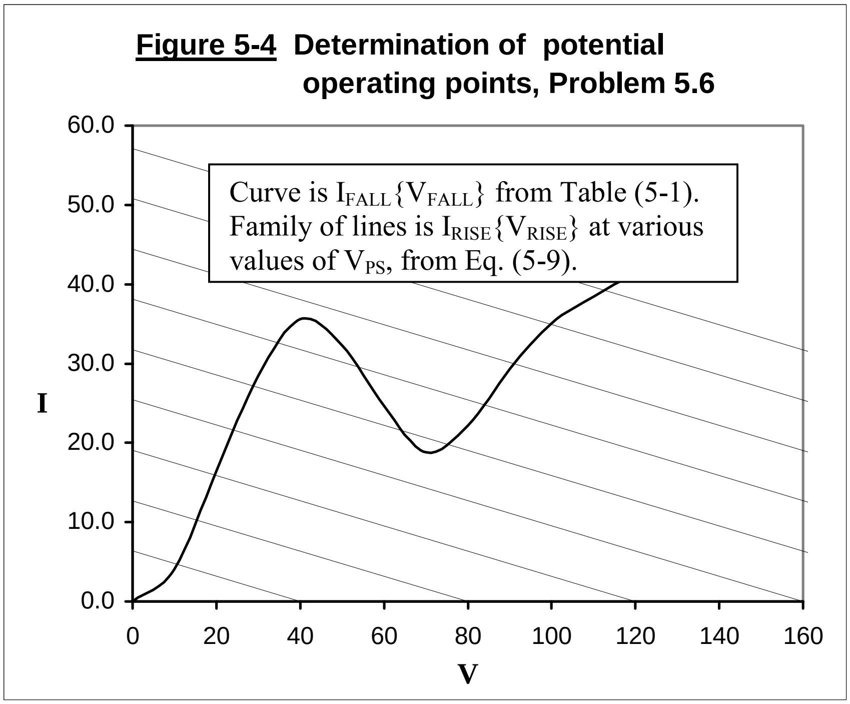
V_D	$I_D\{V_D\}$	$I_B\{V_D\}$	$I_C\{V_D\}$	$(I_B + I_C + I_D)\{V_D\} = (I_{FALL})\{V_{FALL}\}$
0	0	0	0	0
10	2.6	0.8	0.6	4.0
20	13.7	1.6	1.2	16.5
30	24.2	2.4	1.8	28.4
40	30	3.1	2.5	35.6
50	25.3	3.9	3.1	32.3
60	16.2	4.7	3.7	24.6
70	9	5.5	4.3	18.8
80	11	6.3	4.9	22.2
90	16.7	7.1	5.5	29.3
100	21	7.9	6.1	35.0
110	23	8.7	6.7	38.4
120	24.5	9.4	7.4	41.3
130	25.5	10.2	8.0	43.7
140	27	11.0	8.6	46.6

Table 5-1 Calculation of $(I_{FALL})\{V_{FALL}\}$ coordinates

Problem 5.6 cont.

- On Figure (5-4), plot $I_{FALL}\{V_{FALL}\}$ coordinates from Table (5-1).

- On Figure (5-4), use Eq. (5-9) to plot $I_{RISE}\{V_{RISE}\}$ curves at various values of V_{PS}. Cover the range V_{PS} = 40 to 400 volts in increments of 40 volts. Note from Eq. (5-9) that, on each curve, V_{PS} is equal to the value of V_{RISE} at I_{RISE} = 0.



- Note that intersections in Figure (5-4) are potential operating points.

With regard to stability at potential operating points, note the following:

- Stability can be determined by inspection of Figure (5-4). As indicated by Criterion (5-1), operation at an intersection is unstable if the slope of $I_{RISE}\{V_{RISE}\}$ is greater than the slope of $I_{FALL}\{V_{FALL}\}$. (Since both slopes are negative, the greater slope is *less* steep. Therefore, intersections in Figure (5-4) are unstable if the $I_{FALL}\{V_{FALL}\}$ curve is *steeper* than the $I_{RISE}\{V_{RISE}\}$ curve.)

Problem 5.6 cont.

- Intersections throughout most of the negative slope region of $I_{FALL}\{V_{FALL}\}$ are unstable.

- All unstable intersections are in the negative slope region of $I_{FALL}\{V_{FALL}\}$.

- All unstable intersections are on $I_{RISE}\{V_{RISE}\}$ lines that have 3 intersections. Only the middle intersections are unstable.

- Because the unstable intersections are middle intersections, a positive perturbation would cause operation to shift to the higher voltage intersection, and a negative perturbation would cause operation to shift to the lower voltage intersection. Since operation is stable at both the higher and lower voltage intersections, the system would remain at either intersection.

Solution

The solution of Problem 5.6 requires that $I_{PS}\{V_{PS}\}$ be determined over the power supply range of 0 to 400 volts. Coordinates of $I_{PS}\{V_{PS}\}$ are obtained in the following manner:

- Inspect Figure (5-4) to determine the values of V_{RISE} and I_{RISE} at intersections.

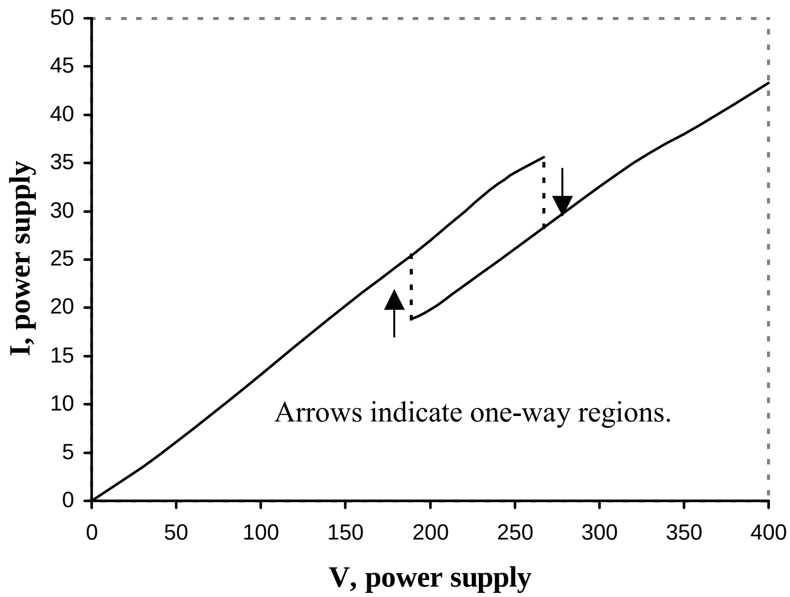
- Substitute the intersection values of V_{RISE} and I_{RISE} in Eq. (5-9) to determine (V_{PS},I_{RISE}) coordinates at intersections.

- Note that I_{PS} and I_{RISE} are equal, and therefore V_{PS},I_{RISE} = V_{PS},I_{PS}.

- Plot the (V_{PS},I_{PS}) coordinates of the stable intersections. Do not plot the coordinates of unstable intersections because the system automatically leaves unstable intersections, and goes to stable intersections, where it remains.

Figure (5-5) is the desired solution—a description of system performance $I_{PS}\{V_{PS}\}$ over the power supply range 0 to 400 volts.

Figure 5-5 **System performance, Problem 5.6**



5.7 How to eliminate hysteresis

Figure (5-5) indicates that the system in Problem 5.6 exhibits pronounced hysteresis when the power supply delivers between 190 and 270 volts. Assuming that the behavior of the nonlinear component cannot be altered, the hysteresis can be eliminated by modifying Components A and E. The required modification can be determined by noting the following:

- The hysteresis in Figure (5-5) results from the multi-valued solutions in Figure (5-4), and therefore the hysteresis would be eliminated if all solutions were single-valued.

- All solutions would be single-valued if the slope of the $I_{RISE}\{V_{RISE}\}$ lines were more negative than the most negative slope region of $(I_{FALL})\{V_{FALL}\}$.

- The slope of the $I_{RISE}\{V_{RISE}\}$ lines could be made more negative by altering the electrical behavior of Components A and E.

- The most negative slope of $(I_{FALL})\{V_{FALL}\}$ is –0.77 amps/volt. The slope of the $I_{RISE}\{V_{RISE}\}$ lines is –0.16 amps/volt, obtained by differentiation of Eq. (5-9).

- The slope of the $I_{RISE}\{V_{RISE}\}$ lines can be decreased to a value less than –0.77 amps/volt by modifying Components A and E so that the sum of the 1.8 in Eq. (5-2) and the 4.5 in Eq. (5-5) is decreased by a factor less than (.16/0.77) = 0.21. This modification ensures that all potential operating points are stable, and therefore the modified system will operate without hysteresis throughout its normal operating range of 0 to 400 volts.

5.8 Stability analysis of an electrical system—Problem 5.8

Problem 5.8 differs from Problem 5.6 in that the system instability results in undamped oscillation as well as hysteresis. It should be noted that undamped oscillation results in spite of a power supply that delivers a constant emf.

Problem statement

Describe the performance of the system in Figure (5-6) over the range 0 to 140 volts.

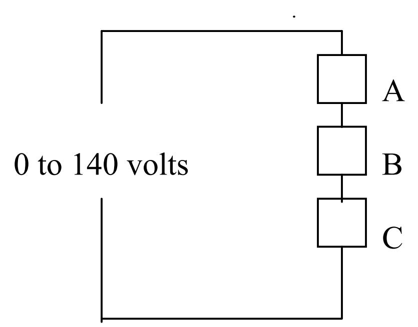


Figure 5-6 **Electric Circuit in Problem 5.8**

Problem 5.8 cont.

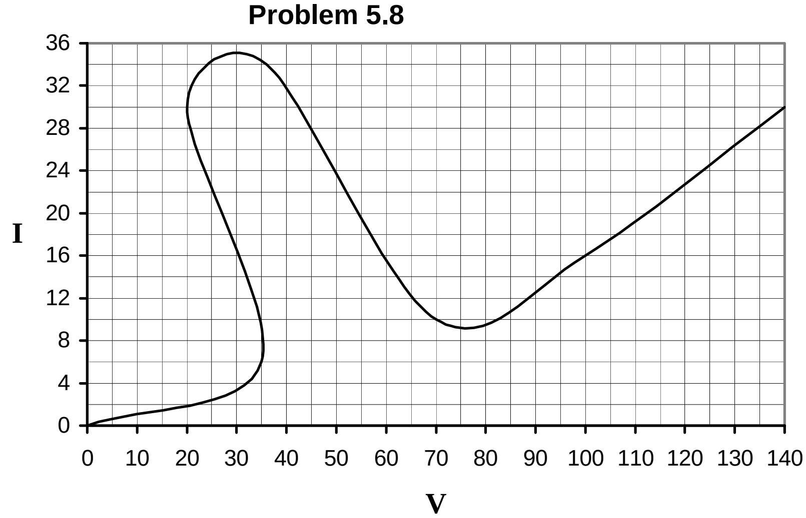
Given

The electrical behavior of Components A and B is given by Eqs. (5-10) and (5-11). The electrical behavior of Component C is given by Figure (5-7).

$$V_A = .85\ I_A \tag{5-10}$$

$$V_B = 1.45\ I_B \tag{5-11}$$

Figure 5-7 Electrical behavior of Component C, Problem 5.8



Determination of potential operating points

Uncouple the circuit so that the FALL subsystem contains only Component C, and the RISE subsystem contains the remainder of the system, including the power supply.

Inspect Figure (5-6) and note that:

$$I_A = I_B = I_C \tag{5-12}$$

$$V_{FALL} = V_C \tag{5-13}$$

$$V_{RISE} = V_{PS} - V_A - V_B \tag{5-14}$$

Problem 5.8 cont.

- Write an equation for $I_{RISE}\{V_{RISE}\}$ by substituting Eqs. (5-10) and (5-11) in Eq. (5-14), and using Eq. (5-12).

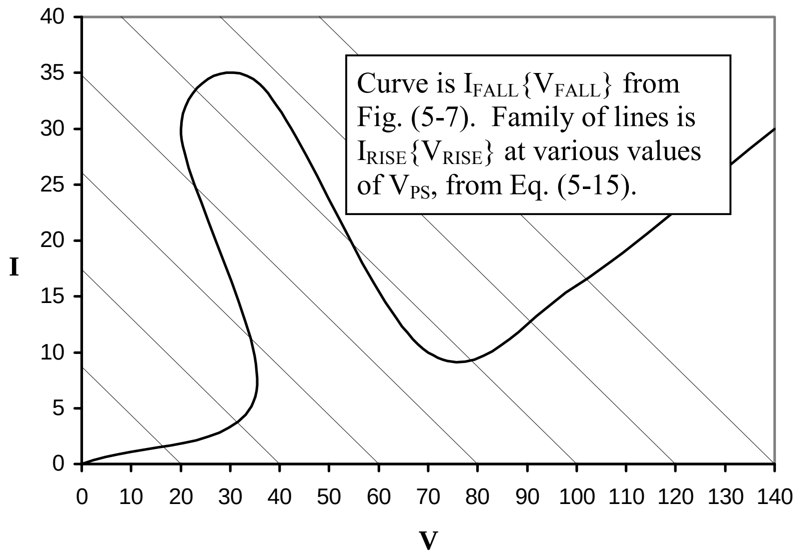
$$V_{RISE} = V_{PS} - .85 I_{RISE} - 1.45\, I_{RISE} \qquad (5\text{-}15a)$$

$$\therefore I_{RISE} = 0.435(V_{PS} - V_{RISE}) \qquad (5\text{-}15b)$$

- Note that Figure (5-7) describes both $I_C\{V_C\}$ and $I_{FALL}\{V_{FALL}\}$.

- Plot $I_{FALL}\{V_{FALL}\}$ from Figure (5-7) on Figure (5-8). Also on Figure (5-8), use Eq. (5-15) to plot $I_{RISE}\{V_{RISE}\}$ curves at various values of V_{PS}. Cover the range V_{PS} = 0 to 140 volts in increments of 20 volts. Note from Eq. (5-15) that, on each curve, V_{PS} is equal to the value of V_{RISE} at $I_{RISE} = 0$.

Figure 5-8 **Determination of potential operating points, Problem 5.8**



- Note that intersections of $I_{FALL}\{V_{FALL}\}$ and $I_{RISE}\{V_{RISE}\}$are potential operating points. Also note that operation is stable at some intersections, and unstable at others.

Problem 5.8 cont.

Stability at intersections

With regard to stability at the intersections in Figure (5-8), note the following:

- In Figure (5-8), the electrical behavior of the FALL subsystem includes a maximum and a minimum in $I_{FALL}\{V_{FALL}\}$, *and* a maximum and minimum in $V_{FALL}\{I_{FALL}\}$.
- The maximum and minimum in $I_{FALL}\{V_{FALL}\}$ occur at (I,V) coordinates of (35,30) and (9,76). The maximum and minimum in $V_{FALL}\{I_{FALL}\}$ occur at (V, I) coordinates (35,7) and (20,30).
- Note that V_{PS} is constant along $I_{RISE}\{V_{RISE}\}$ lines, and therefore the value of V_{PS} on each line can be determined by inspection of Figure (5-8), since Eq. (5-15) indicates that $V_{PS} = V_{RISE}\{I_{RISE} = 0\}$).
- Note in Figure (5-8) that, when V_{PS} is greater than 50 volts and less than 87 volts, $I_{RISE}\{V_{RISE}\}$ intersects $I_{FALL}\{V_{FALL}\}$ in the region between the maximum and minimum in $V_{FALL}\{I_{FALL}\}$.
- Note that, when $I_{RISE}\{V_{RISE}\}$ intersects $I_{FALL}\{V_{FALL}\}$ in the region between the maximum and minimum in $V_{FALL}\{I_{FALL}\}$, only a single intersection results, and it is unstable. Since there is only one intersection, the system cannot “find” a stable intersection, and it remains in an unstable condition.

Behavior at unstable, single intersections

To determine the system behavior that results from a single, unstable intersection, refer to Figure (5-8), and suppose that the system is initially at $V_{PS} = 80$:

- The system suddenly receives a small, positive perturbation in V.
- The positive perturbation causes V to increase because, in the perturbed condition, I_{RISE} is greater than I_{FALL}.
- When V increases to the maximum in $V_{FALL}\{I_{FALL}\}$ at (35,7), the mismatch between I_{RISE} and I_{FALL} causes a step increase to (35,34), since it is the only operating point at V incrementally greater than 35.

Problem 5.8 cont.

- At (35,34), the mismatch between I_{RISE} and I_{FALL} causes V to decrease to the minimum in $V_{FALL}\{I_{FALL}\}$ at (20,30).

- At (20,30), the mismatch between I_{RISE} and I_{FALL} causes a step decrease to (20,2).

- At (20,2), the mismatch between I_{RISE} and I_{FALL} causes V to increase to the maximum in $V_{FALL}\{I_{FALL}\}$ at (35,7), and the cycle repeats.

Note in Figure (5-8) that, when V_{PS} is greater than 50 volts and less than 87 volts, a single, unstable intersection results. Therefore, when V_{PS} is between 50 and 87 volts, I_C and V_C endlessly traverse the loop shown in Figure (5-9).

Also note in Figure (5-8) that, when V_{PS} is greater than 92 volts and less than 113 volts, three intersections result. As in Problem 5.6, the middle intersection is unstable, and the instability results in hysteresis.

In summary, inspection of Figure (5-8) indicates that the system exhibits:

- Undamped oscillation when $I_{RISE}\{V_{RISE}\}$ intersects $I_{FALL}\{V_{FALL}\}$ in the region between the maximum and minimum in $V_{FALL}\{I_{FALL}\}$. This occurs when the power supply delivers 50 to 87 volts, and results in the endless loop shown in Figure (5-9)..

- Hysteresis when $I_{RISE}\{V_{RISE}\}$ intersects $I_{FALL}\{V_{FALL}\}$ in the region between the maximum and minimum in $I_{FALL}\{V_{FALL}\}$. This occurs when the power supply delivers 92 to 113 volts.

Solution

The solution of Problem 5.8 is a chart of $I_{PS}\{V_{PS}\}$. Coordinates of $I_{PS}\{V_{PS}\}$ are obtained from $I_{RISE}\{V_{RISE}\}$ intersections in Figure (5-8) by noting that $I_{PS} = I_{RISE}$, and $V_{PS} = V_{RISE}\{I_{RISE} = 0\}$. Unstable intersections are *not* plotted.

Figure (5-10) is the solution of Problem 5.8. It describes the system performance over the power supply range of 0 to 140 volts.

Figure 5-9 **Endless loop that results when power supply delivers 50 to 87 volts, Problem 5.8**

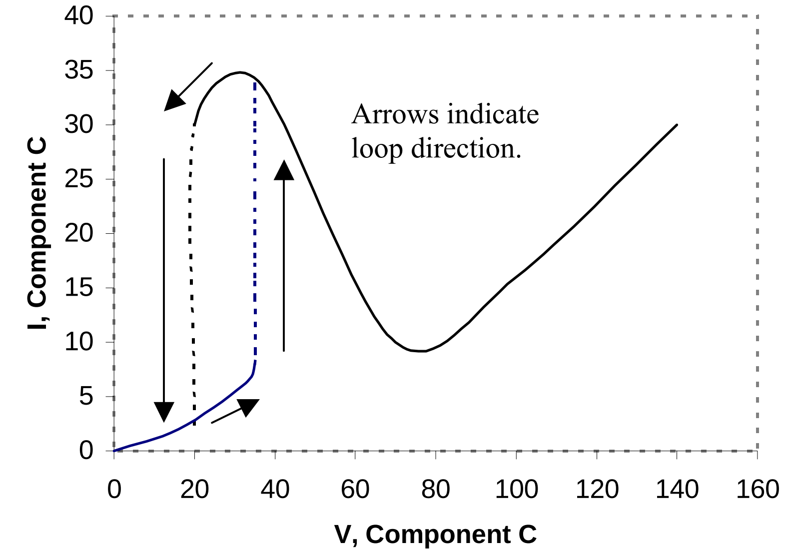
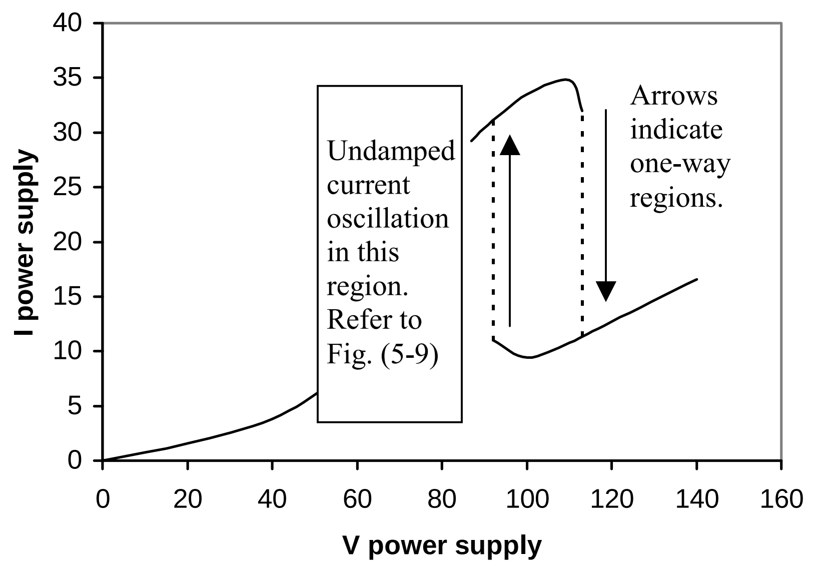


Figure 5-10 **Performance of system in Problem 5.8**



5.9 How to eliminate undamped oscillations

Assuming that the behavior of Component C cannot be altered, the undamped oscillations noted in Problem 5.8 could be eliminated by modifying the system to ensure that all unstable intersections lie on $I_{RISE}\{V_{RISE}\}$ lines that make three intersections with $I_{FALL}\{V_{FALL}\}$.

Note in Figure (5-8) that, if the $I_{RISE}\{V_{RISE}\}$ lines were steeper than any point on $V_{FALL}\{I_{FALL}\}$ in the region between the maximum and minimum in $V_{FALL}\{I_{FALL}\}$, all single solutions would be replaced by triple solutions, as desired.

In the region between the maximum and the minimum in $V_{FALL}\{I_{FALL}\}$, the largest slope is -1.13 amps/volt. Therefore, if the system were modified so that the slope of the $I_{RISE}\{V_{RISE}\}$ lines were ≤ -1.13 amps/volt, the singular solutions would be replaced by triple solutions, and the undamped oscillations would be replaced by hysteresis.

The slope of the $I_{RISE}\{V_{RISE}\}$ lines is determined by the behavior of Components A and B. To attain the desired slope, Components A and B must be modified so that the 0.435 in Eq. (5-15) becomes equal to or greater than 1.13.

If Components A and B were modified as required, the system performance would be affected in the following ways:

- Undamped oscillatory behavior would be eliminated. It would be replaced by hysteresis at power supply voltages in the vicinity of 35 volts. The extent of the hysteresis would depend on the behavior of the modified Components A and B.

- The hysteresis that occurred in the original design (between 92 and 113 volts) would be eliminated, since the steepest slope between the maximum and minimum in $I_{FALL}\{V_{FALL}\}$ is $-.85$ amps/volt.

5.10 Conclusions

Stability analysis of electrical systems is simple and effective using behavior methodology.

Chapter 6

Inductance, Capacitance, and Summary

6 Introduction

Preceding chapters appraise electrical resistance *V/I*, and demonstrate that *V/I* should be abandoned in favor of resistive electrical behavior V{I}. But in a larger sense, the preceding chapters demonstrate that ratios of primary parameters should *generally* be abandoned in favor of behavior methodology. Electrical resistance *V/I* is merely the example chosen for illustration.

In the new engineering, *all* ratios that combine primary parameters are abandoned in favor of behavior methodology. In this book, it would hardly be possible to rigorously demonstrate that each ratio that combines primary parameters is unnecessary and undesirable. Some are appraised at length, some are appraised by analogy, and some are not appraised at all.

Four ratios of primary parameters are appraised at length:

- Electrical resistance
- Heat transfer coefficient
- Material modulus
- Fluid friction factor

They are taken from different branches of engineering in order to demonstrate that ratios that combine primary parameters should *generally* be abandoned in favor of behavior methodology.

In this chapter, electrical inductance and electrical capacitance are appraised by analogy to electrical resistance. Electrical resistance, electrical inductance, and electrical conductance are ratios of primary parameters. All are unnecessary and undesirable. All are abandoned in the new engineering.

6.1 The resistance analog of inductance and capacitance

The close analogy between electrical resistance, electrical induction, and electrical capacitance is seen by noting that

- Resistance is *V/I*, symbol *R*, dimension "ohms" (actually volts/amp).

- Inductance is *V/(dI/dt)*, symbol *L*, dimension "henries" (actually volt-secs/amp).

- Capacitance is *q/V*, symbol C, dimension "farads" (actually amp-secs/volt).

In this chapter, the resistance analog is used to appraise electrical "inductance" and electrical "capacitance".

6.2 Appraisal of electrical resistance

In preceding chapters, appraisal of electrical resistance established that:

- Electrical resistance is the ratio *V/I*.

- The ratio *V/I* is assigned the name electrical "resistance", the symbol *R*, and the dimension "ohms".

- Since electrical resistance is *V/I*, the actual dimension is volts/amp. However, "ohms" was made a synonym of volts/amp, and is generally used in place of volts/amp. (Just as "hertz" was made a synonym of "cycles per second", and is used in place of cycles per second.)

- Electrical resistance *V/I* is unnecessary because problems can be solved without combining *V* and *I* in the ratio *V/I*.

- Electrical resistance *V/I* is undesirable because combining *V* and I greatly complicates the solution of nonlinear problems in general.

- Electrical resistance *V/I* should be abandoned because it is unnecessary and undesirable.

- Electrical resistance *V/I* should be replaced by resistive electrical behavior V{I} in order to keep the primary parameters separate.

- Electrical resistance methodology should be replaced by resistive electrical behavior methodology in order that problems may be solved with the primary parameters separated.

6.3 Appraisal of electrical inductance

The following appraisal of electrical "inductance" is based on the close analogy between it and electrical resistance:

- Electrical inductance is the ratio *V/(dI/dt)*.

- The ratio *V/(dI/dt)* is assigned the name electrical "inductance", the symbol *L*, and the dimension "henries".

- Since electrical inductance is *V/(dI/dt)*, the actual dimension is volt-secs/amp. However, "henries" was made a synonym of volt-secs/amp, and is generally used in place of volt-secs/amp. (Just as "hertz" was made a synonym of "cycles per second", and is generally used in place of cycles per second.)

- Electrical inductance *V/(dI/dt)* is unnecessary because problems can be solved without combining *V* and *dI/dt* in the ratio *V/(dI/dt)*.

- Electrical inductance *V/(dI/dt)* is undesirable because combining *V* and *dI/dt* greatly complicates the solution of nonlinear problems in general.

- Electrical inductance *V/(dI/dt)* should be abandoned because it is unnecessary and undesirable.

- Electrical inductance *V/(dI/dt)* should be replaced by inductive electrical behavior V{dI/dt} in order to keep the primary parameters separate.

- Electrical inductance methodology should be replaced by inductive electrical behavior methodology in order that problems may be solved with the primary parameters separated.

6.4 Appraisal of electrical capacitance

The following appraisal of electrical "capacitance" is based on the close analogy between it and electrical resistance:

- Electrical capacitance is the ratio *q/V* where *q* is electric charge.
- The ratio *q/V* is assigned the name electrical "capacitance", the symbol *C*, and the dimension "henries".
- Since electrical capacitance is *q/V*, the actual dimension of capacitance is amp-secs/volt. However, "farads" was made a synonym of amp-secs/volt, and is generally used in place of amp-secs/volt. (Just as "hertz" was made a synonym of "cycles per second", and is generally used in place cycles per second.)
- Electrical capacitance *q/V* is unnecessary because problems can be solved without combining *q* and *V* in the ratio *q/V*.
- Electrical capacitance *q/V* is undesirable because combining *q* and *V* greatly complicates the solution of nonlinear problems in general.
- Electrical capacitance *q/V* should be abandoned because it is unnecessary and undesirable.
- Electrical capacitance *q/V* should be replaced by capacitive electrical behavior *q{V}* in order to keep the primary parameters separate.
- Electrical capacitance methodology should be replaced by capacitive electrical behavior methodology in order that problems may be solved with the primary parameters separate.

6.5 A summary of electrical science in the new engineering

Electrical science in the new engineering is summarized as follows:

- The parameters resistance, inductance, and capacitance are abandoned.

- The words resistance, inductance, and capacitance are abandoned.

- The "dimensions" ohms, henries, and farads are abandoned.

- The word "resistive" is used to indicate electrical behavior described by V{I}.

- Resistive electrical phenomena are described, analyzed, and predicted using resistive electrical behavior V{I}. For example, the equation $R = 3$ ohms is replaced by E = 3I. Similarly, the equation $R = 4.2\ I^{1.5}$ ohms is replaced by $V = 4.2\ I^{2.5}$.

- The word "inductive" is used to indicate electrical behavior described by V{dI/dt}.

- Inductive electrical phenomena are described, analyzed, and predicted using inductive electrical behavior V{dI/dt}. For example, the equation $L = 1.2$ henries is replaced by V = 1.2 dI/dt.

- The word "capacitive" is used to indicate electrical behavior described by q{V}.

- Capacitive electrical phenomena are described, analyzed, and predicted using capacitive electrical behavior q{V}. For example, the equation C = 2.5 farads is replaced by q = 2.5V.

- In electrical analyses, q, V, and I and their derivatives are *separate* and *explicit*.

Chapter 7

Example problems that illustrate heat transfer analysis using behavior methodology

7 Introduction

This chapter contains example problems that illustrate heat transfer analysis using behavior methodology—ie methodology that focuses on the behavior of the primary parameters—ie methodology in which the primary parameters q and ΔT are *separate* and *explicit*. The problems include proportional and nonlinear phenomena, and demonstrate that heat transfer analysis is simple and direct using behavior methodology.

In Chapter 8, the problems in this chapter are restated (but not solved) using heat transfer coefficient methodology. The reader is encouraged to solve the problems using coefficient methodology in order to gain a first hand appreciation of the simplicity that results from behavior methodology.

The problems in Chapters 7 and 8 demonstrate that:

- Proportional problems can be solved in a simple and direct manner using either behavior methodology or coefficient methodology.

- Nonlinear problems that must be solved in an indirect manner using coefficient methodology can be solved in a direct and much simpler manner if behavior methodology is used.

7.1 Electrical analog of heat transfer

In the new engineering, the electrical analog of heat transfer differs from the analog in conventional engineering because I{V} replaces *R* and Ohm's law, and q{ΔT} replaces *h* and "Newton's law of cooling". In the new engineering, the electrical analog is described by the following:

Heat transfer	**Electrical analog**
Temperature difference, ΔT	Electromotive force, V
Heat flux, q	Electric current, I
$q = f\{\Delta T\}$	$I = f\{V\}$

The analogy between electrical phenomena and heat transfer phenomena is so close that, in order to transform the electrical problems in Chapter 2 to heat transfer problems, little more is required than substituting q for I, and ΔT for V.

7.2 Heat transfer analysis

In conventional engineering and in the new engineering, heat transfer analysis often concerns one or more of the following:

- Determine the heat flux from a heat source to a heat sink.
- Determine the temperature profile from a heat source to a heat sink.
- Determine the total heat flow rate Q by integrating the local heat flux over the heat transfer surface.

Integration in the new engineering is the same as integration in conventional engineering. Therefore little space in this book is devoted to integration.

In this chapter, problems that concern the determination of heat flux and temperature profile are solved using heat transfer behavior methodology. In Chapter 8, the reader is encouraged to solve these same problems using heat transfer coefficient methodology.

7.3 Using behavior methodology to describe the relationship between q and dT/dx

In conventional engineering, the conductive heat transfer behavior of materials is described by thermal conductivity *k*, defined by Eq. (7-1).

$$q_{COND} = k\{T\}dT/dx \qquad (7\text{-}1)$$

Note that:

- If *k* is viewed as the ratio *q/(dT/dx)*, it unconditionally describes conductive behavior in a global way, since this ratio can describe behavior that is proportional or linear or nonlinear.

- If *k* is viewed as the dimensioned proportionality constant between *q* and d*T/dx*, it can describe behavior in a global way *only* if *q* is proportional to *dT/dx* for *all* practical materials.

- Because all practical materials currently exhibit proportional conductive behavior, *k* is now global in the form of the ratio *q/(dt/dx)*, *and* in the form of the dimensioned proportionality constant between *q* and *dT/dx*.

- When materials have been developed that exhibit nonlinear conductive behavior:
  - *k* in the form of the ratio *q/(dT/dx)* will still be a global parameter.
  - *k* in the form of the dimensioned proportionality constant between *q* and *dT/dx* will no longer be a global parameter.
  - *k* will have to be viewed in general as the ratio *q/(dT/dx)*.

In the new engineering, Eq. (7-2) is unconditionally a global description of behavior, since *f*{dT/dx} can describe behavior that is proportional or linear or nonlinear.

$$q_{COND} = a\{T\}f\{dT/dx\} \qquad (7\text{-}2)$$

Because all practical materials currently exhibit proportional conductive behavior, Eq. (7-2) can be written in the specific form of Eq. (7-3):

$$q_{COND} = K\{T\}(dT/dx) \qquad (7\text{-}3)$$

It is important to note that:

- K{T} is a pure number. It is *not* the numerical value of a parameter in specified dimensions, and it is *not* a parameter in unspecified dimensions.

- K{T} is the proportionality constant between q_{COND} and dT/dx. Therefore its value is partially determined by the dimensions specified for q_{COND} and dT/dx.

- Because all practical materials currently exhibit proportional conductive behavior, Eq. (7-3) now applies globally. It will cease to apply globally when materials are developed that exhibit nonlinear conductive behavior. However, it will still be the specific form used to describe proportional conductive behavior.

In the remainder of this text, it is assumed that all materials exhibit proportional conductive behavior. Therefore Eq. (7-3) applies globally.

K{T} is readily determined from *k{T}* in the following manner:

- Express the given value of *k{T}* in the dimensions specified for q and dT/dx.

- Note that K{T} and *k{T}* are numerically equal if the dimensions of *k* are the same as those specified for q and dT/dx.

7.4 Using behavior methodology to describe the relationship between q and ΔT

In the new engineering, the relationship between q and ΔT is described with q and ΔT *separate* and *explicit* in the behavior form q{ΔT} or ΔT{q}—ie in the form q = *f*{ΔT} or ΔT = *f*{q}.

The behavior form is used for:

- Equations or charts that describe the heat transfer behavior of fluid boundary layers in a general way, and apply to different geometries, fluids, flow rates, etc. Eq. (7-4) is an example.

$$q = .023(K/D)N_{Re}^{.8}N_{Pr}^{.4}\,\Delta T \qquad (7\text{-}4)$$

- Equations or charts that describe the heat transfer behavior of a specific boundary layer or wall. Eqs. (7-5) and (7-6), and Figure (7-1) are examples.

$$q = 5.8\ \Delta T \qquad (7\text{-}5)$$

$$q = 2.5\ \Delta T^{1.33} \qquad (7\text{-}6)$$

- Equations or charts that describe the overall heat transfer behavior of a series of specific boundary layers and walls in a specific application. Eqs. (7-5) and (7-6), and Figure (7-1), are examples.

Eqs. (7-4C) to (7-6C) and Figure (7-1C) are *identical* to Eqs. (7-4) to (7-6) and Figure (7-1). They differ only in form—coefficient form vs behavior form. (Note that the identifying numbers are the same except that C has been added to indicate coefficient form.)

$$h = .023(k/D)\ Re^{.8}\ Pr^{.4}\ \text{Btu/hrft}^2\text{F} \qquad (7\text{-}4C)$$

$$h = 5.8\ \text{Btu/hrft}^2\text{F} \qquad (7\text{-}5C)$$

$$h = 2.5\ \Delta T^{0.33}\ \text{Btu/hrft}^2\text{F} \qquad (7\text{-}6C)$$

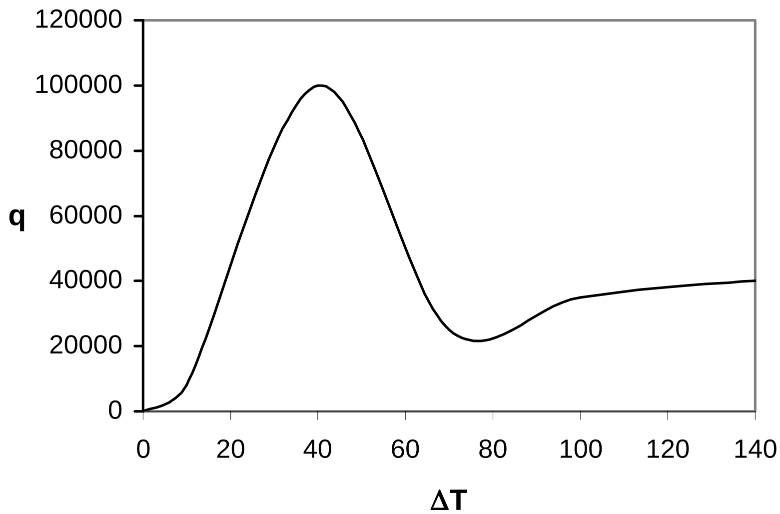
The equations and figures above describe the relationship between q and ΔT. Notice that the relationship between q and ΔT is more clearly revealed in the behavior form, particularly when the relationship is highly nonlinear.

7.5 Parameter groups in the new heat transfer

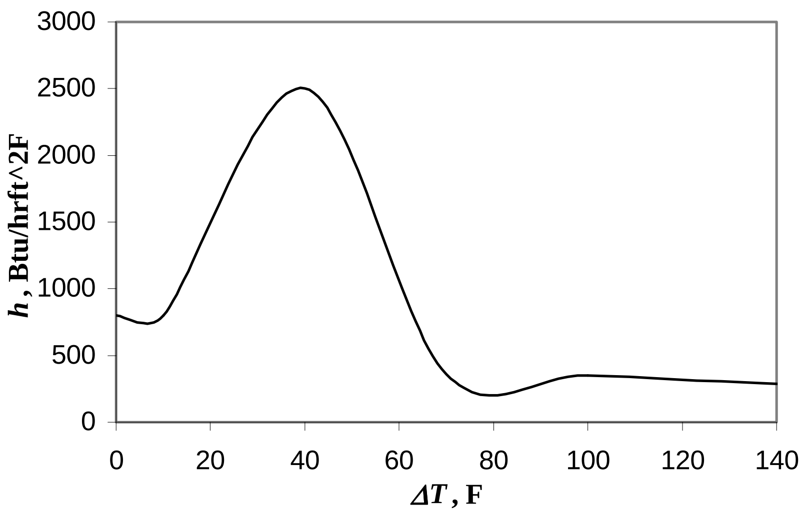
In order that the primary parameters q and ΔT may remain separate and explicit, parameter groups are used somewhat differently in the new heat transfer:

- Parameter groups that include *both* q and ΔT *are not used in any form* because they combine q and ΔT. For example, Nusselt number and Stanton number contain both q and ΔT (since they contain *h*, and *h* is $q/\Delta T$). Therefore Nusselt number and Stanton number are not used in any form.

<u>Figure 7-1</u> Example of heat transfer behavior q{ΔT}



<u>Figure 7-1C</u> Heat transfer coefficient form of Figure 7-1



- Parameter groups that include *either* q or ΔT are used, but *only* with the individual parameters shown explicitly. For example, the individual parameters are implicit in N_{Re}, and explicit in (DG/μ).

- Parameter groups that include *neither* q nor ΔT are used in both explicit and implicit forms. For example, Reynolds number contains neither q nor ΔT. It is used in both explicit and implicit forms.

7.6 How to transform heat transfer coefficient correlations to q{ΔT} correlations

Heat transfer coefficient correlations are transformed to heat transfer behavior correlations in the following way:

- Substitute $q/\Delta T$ for *h* and K for *k*.

- Separate q and ΔT.

For example, Eq. (7-7) is a generalized heat transfer coefficient correlation for fluid boundary layers.

$$N_{Nu} = .023\, N_{Re}^{.8}\, N_{Pr}^{.4} \tag{7-7}$$

Eq. (7-7) is transformed to a q{ΔT} correlation as follows:

- Note that N_{Nu} is *hD/k*.

- Substitute $q/\Delta T$ for *h* and K for *k* and obtain $N_{Nu} = qD/\Delta TK$.

- Replace N_{Nu} in Eq. (7-7) by $qD/\Delta TK$.

$$(qD/\Delta TK) = .023\, N_{Re}^{.8}\, N_{Pr}^{.4} \tag{7-8}$$

- Separate q and ΔT and rearrange.

$$q = .023\, (K/D)\, N_{Re}^{.8}\, N_{Pr}^{.4}\, \Delta T \tag{7-9}$$

Eq. (7-9) is in the desired form q{ΔT}.

7.7 The behavior form of $UA\Delta T_{LM}$

In conventional heat transfer, the overall heat transfer coefficient is assigned the symbol *U*. In heat exchanger analysis, if *U* is independent of location, total heat flow Q_{TOTAL} is calculated from Eq. (7-10). (Subscript LM refers to log mean.)

$$Q_{TOTAL} = U\,A\,\Delta T_{LM} \qquad (7\text{-}10)$$

Eq. (7-10) is converted to behavior form by noting that

$$U\,\Delta T_{LM} = \mathrm{q}\{\Delta \mathrm{T}_{\mathrm{LM}}\} \qquad (7\text{-}11)$$

Combining Eqs. (7-10) and (7-11) gives Eq. (7-12):

$$\therefore \mathrm{Q}_{\mathrm{TOTAL}} = \mathrm{q}\{\Delta \mathrm{T}_{\mathrm{LM}}\}\ \mathrm{A} \qquad (7\text{-}12)$$

Eq. (7-12) is the behavior form of Eq. (7-10). It is valid only if q is everywhere proportional to ΔT, *and* the value of the proportionality constant is independent of location.

7.8 How to determine $q\{\Delta T_{WALL}\}$ equations

$q\{\Delta T_{WALL}\}$ equations can be obtained in the following way for materials that exhibit *proportional* conductive heat transfer behavior—ie for all currently practical materials:

- Obtain *k{T}* from conventional engineering literature, and convert it to K{T} in the manner described in Section 7.3.

- Make the simplifying assumption that K is independent of T. (This assumption simplifies the integration in the next step.)

- Substitute K in Eq. (7-3). Integrate Eq. (7-3) and obtain Eq. (7-13).

$$\mathrm{q}_{\mathrm{WALL}} = (\mathrm{K/t})\ \Delta \mathrm{T} \qquad (7\text{-}13)$$

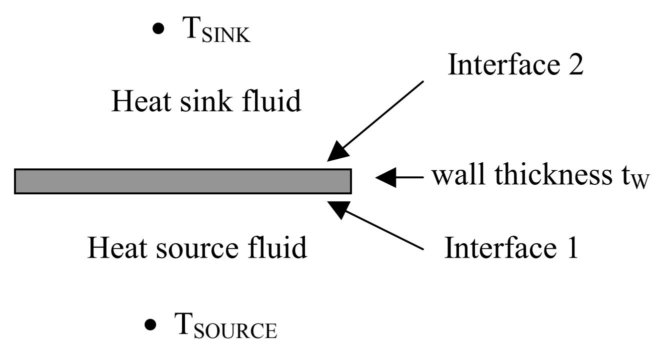
- Substitute in Eq. (7-13), and obtain a specific q{ΔT} equation for the wall.

$$\mathrm{q}_{\mathrm{WALL}} = \mathrm{a}\ \Delta \mathrm{T} \qquad (7\text{-}14)$$

Eq. (7-14) results *only* if q{dT/dx} is proportional. If q{dT/dx} is non-linear, the result of integration will be more complex than Eq. (7-14).

7.9 Heat transfer analysis using behavior methodology

A typical problem in heat transfer analysis is to determine the heat flux that would result from connecting a heat source fluid to a heat sink fluid. The source fluid and sink fluid are often connected through a wall as shown in Figure (7-2).



<u>Figure 7-2</u> Typical heat transfer configuration

The heat flux in the Figure (7-2) configuration is determined using the following behavior methodology:

The problem statement specifies:

- Geometry of the component
- Identity of source fluid, sink fluid, and wall material
- Temperatures and flow rates of source fluid and sink fluid
- Thickness of wall material.

The following information is obtained from heat transfer literature:

- Generalized q{ΔT} correlations for the source fluid boundary layer and the sink fluid boundary layer. (Correlations in the current literature are generally in *h* form. They are transformed to q{ΔT} form in the manner described above.)

- Conductive heat transfer behavior. (In conventional engineering literature, conductive heat transfer information is in thermal conductivity form. It is converted to behavior form in the manner described in Section 7.3.)

The analysis is performed as follows:

- Obtain a specific $q\{\Delta T\}$ equation for each boundary layer by evaluating the generalized correlations at the specified geometry, flow rate, temperature, etc.

- Obtain a specific $q\{\Delta T\}$ equation for the wall using Eq. (7-13).

- Note that the total temperature difference between source and sink is the sum of the individual temperature differences across the boundary layers and the wall. In other words, note that Eq. (7-15) describes the configuration in Figure (7-2).

$$\Delta T_{TOTAL} = T_{SOURCE} - T_{SINK} = \Delta T_1 + \Delta T_2 + \Delta T_{WALL} \qquad (7\text{-}15)$$

- Obtain an equation or graph that relates q and ΔT_{TOTAL} by using the specific $q\{\Delta T\}$ equations to substitute q functions for ΔT_1, ΔT_2, and ΔT_{WALL} in Eq. (7-15).

- Solve the equation for q.

The above procedure applies for any number of walls and boundary layers in series. The only change in the above is that the number of terms in Eq. (7-15) increases as the number of heat transfer elements is increased.

7.10 Heat transfer behavior methodology—Problem 7.10

Problem 7.10 concerns the determination of heat flux and temperature profile. All heat transfer elements in the problem exhibit proportional behavior, and therefore the solution of the problem is simple and direct using either coefficient methodology or behavior methodology.

Problem statement

What is the heat flux in the Figure (7-3) configuration? What is the wall temperature at each interface?

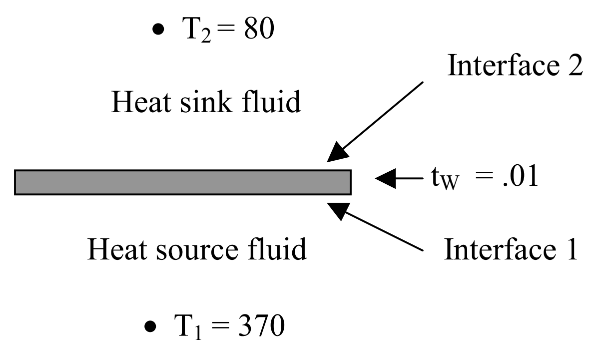


Figure 7-3 Heat transfer configuration, Problem 7.10

Given

- Equipment drawings
- Identity of source fluid, sink fluid, and wall material
- Flow rate of source fluid and sink fluid
- Boundary layer heat transfer is described by Eq. (7-16), obtained from the literature of conventional engineering.

$$N_{Nu} = .023\ N_{Re}^{.8}\ N_{Pr}^{.4} \qquad (7\text{-}16)$$

<u>Problem 7.10 cont.</u>

- The thermal conductivity of the wall is obtained from the literature of conventional engineering.

$$k_W = 14.5 \text{ Btu/hrftF} \qquad (7\text{-}17)$$

Analysis

- Transform Eq. (7-16) from a heat transfer coefficient correlation to a heat transfer behavior correlation by substituting ($qD/\Delta TK$) for N_{Nu}, and separating q and ΔT. The result is Eq. (7-18).

$$q = .023\ (K/D)\ N_{Re}^{.8}\ N_{Pr}^{.4}\ \Delta T \qquad (7\text{-}18)$$

(Note that K in Eq. (7-18) is obtained from the literature value of *k* in the manner described in Section 7.3.)

- Obtain a q{ΔT} equation for each boundary layer by evaluating Eq. (7-18) at the given conditions. Assume this was done, and Eqs. (7-19) and (7-20) resulted.

$$q_1 = 158\ \Delta T_1 \qquad (7\text{-}19)$$

$$q_2 = 85\ \Delta T_2 \qquad (7\text{-}20)$$

- Obtain a specific q{ΔT} equation for the wall by determining K_W from k_W in the manner described in Section 7.3, and substituting in Eq. (7-13).

$$q_W = (K_W/t_W)\ \Delta T_W = (14.5/.01)\Delta T_W = 1450\ \Delta T_W \qquad (7\text{-}21)$$

- Obtain an equation that relates q and ΔT_{TOTAL}:
  - Note that the configuration in Figure (7-3) is described by Eqs. (7-22) and (7-23).

$$\Delta T_{TOTAL} = T_1 - T_2 = \Delta T_1 + \Delta T_W + \Delta T_2 \qquad (7\text{-}22)$$

$$q_1 = q_W = q_2 = q \qquad (7\text{-}23)$$

Problem 7.10 cont.

- o Use Eqs. (7-19) to (7-21) to substitute for ΔT_1, ΔT_2, and ΔT_{WALL} in Eq. (7-22). Combine the result with Eq. (7-23).

$$\Delta T_{TOTAL} = (370 - 80) = q/158 + q/85 + q/1450 \qquad (7\text{-}24)$$

- Solve Eq. (7-24) for q. The result is q = 15,400.

- The wall temperature at Interface 1 is determined from Eq. (7-25), using Eq. (7-19) and the calculated value of q.

$$T_{W1} = T_1 - \Delta T1 \qquad (7\text{-}25)$$

$$\therefore T_{W1} = 370 - 15400/158 = 273$$

- The wall temperature at Interface 2 is determined from Eq. (7-26) using Eq. (7-20) and the calculated value of q.

$$T_{W2} = T_2 + \Delta T_2 \qquad (7\text{-}26)$$

$$\therefore T_{W2} = 80 + 15400/85 = 261 \qquad (7\text{-}27)$$

Solution

- The heat flux in Figure (7-3) is 15,400 B/hrft^2.

- The wall temperature at Interface 1 is 273 F.

- The wall temperature at Interface 2 is 261 F.

7.11 Heat transfer behavior methodology—Problem 7.11

Problem 7.11 differs from Problem 7.10 in that the boundary layers exhibit moderately nonlinear behavior. In spite of the nonlinearity, the solution of the problem is simple and direct using behavior methodology. Note in the next chapter that this problem must be solved in an *indirect* (and more difficult) manner if coefficient methodology is used.

Problem statement

What is the heat flux in the Figure (7-4) configuration? What is the wall temperature at each interface?

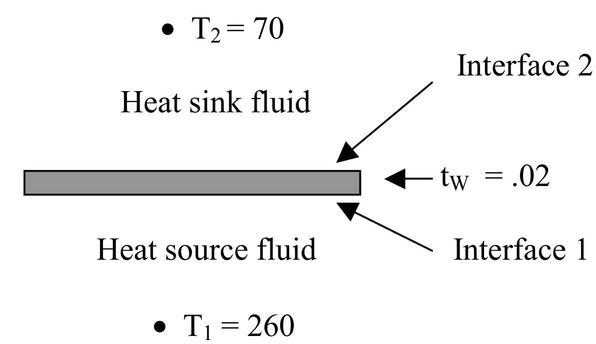


Figure 7-4 Heat transfer configuration, Problem 7.11

Given

- Equipment drawings
- Identity of source fluid, sink fluid, and wall material
- Flow rate of source fluid and sink fluid.
- Boundary layer heat transfer is described by Eqs. (7-28) and (7-29). Both correlations were obtained from the literature of conventional engineering.

$$N_{Nu1} = 0.15\,(N_{Gr1}\,N_{Pr1})^{.33} \qquad (7\text{-}28)$$

$$N_{Nu2} = 0.47\,(N_{Gr2}\,N_{Pr2})^{.20} \qquad (7\text{-}29)$$

Problem 7.11 cont.

- Wall heat transfer is described by Eq. (7-30), obtained from the literature of conventional engineering.

$$k_W = 8.6 \text{ Btu/hrftF} \qquad (7\text{-}30).$$

Analysis

- Transform Eqs. (7-28) and (7-29) from heat transfer coefficient correlations to heat transfer behavior correlations by substituting ($qL/\Delta TK$) for N_{Nu}, and separating q and ΔT.

$$q_1 = 0.15(K/L)(g\beta L^3/\nu^2)^{.33}\, N_{Pr}^{.33}\, \Delta T_1^{1.33} \qquad (7\text{-}31)$$

$$q_2 = 0.37(K/L)(g\beta L^3/\nu^2)^{.20}\, N_{Pr}^{.20}\, \Delta T_2^{1.20} \qquad (7\text{-}32)$$

- Evaluate the parameter groups in Eqs. (7-31) and (7-32) using the given information and the literature. Assume that the literature was consulted, and the following values were obtained:

	(K/L)	**($g\beta L^3/\nu^2$)**	**N_{Pr}**
Fluid 1	.23	360000	1.7
Fluid 2	.31	250000	2.4

- Rewrite Eqs. (7-31) and (7-32) using the parameter group values listed above.

$$q_1 = 2.80\, \Delta T_1^{1.33} \qquad (7\text{-}33)$$

$$q_2 = 1.64\, \Delta T_2^{1.20} \qquad (7\text{-}34)$$

- Obtain a specific q{ΔT} equation for the wall by substituting in Eq. (7-13).

$$q_W = (K/t_W)\, \Delta T_W = (8.6/.02)\, \Delta T_W = 430\, \Delta T_W \qquad (7\text{-}35)$$

Problem 7.11 cont.

- Obtain an equation that relates q and $\Delta T_{TOTAL.}$
    - Obtain Eqs. (7-36) and (7-37) by inspection of Figure (7-4).

$$\Delta T_{TOTAL} = T_1 - T_2 = \Delta T_1 + \Delta T_W + \Delta T_2 \qquad (7\text{-}36)$$

$$q_1 = q_W = q_2 = q \qquad (7\text{-}37)$$

    - Use Eqs. (7-33) to (7-35) to substitute for ΔT_1, ΔT_2, and ΔT_W in Eq. (7-36). Combine the result with Eq. (7-37).

$$\Delta T_{TOTAL} = 260 - 70 = (q/2.80)^{.75} + (q/1.64)^{.833} + (q/430) \qquad (7\text{-}38)$$

- Solve Eq. (7-38) and obtain q = 585.

- The wall temperature at Interface 1 is obtained from Eq. (7-39), using Eq. (7-33) and the calculated value of q.

$$T_{W1} = T_1 - \Delta T_1 \qquad (7\text{-}39)$$

$$\therefore T_{W1} = 260 - (585/2.80)^{.75} = 205$$

- The wall temperature at Interface 2 is obtained from Eq. (7-40), using Eq. (7-34) and the calculated value of q.

$$T_{W2} = T_2 + \Delta T_2 \qquad (7\text{-}40)$$

$$\therefore T_{W2} = 70 + (585/1.64)^{.833} = 204$$

Solution

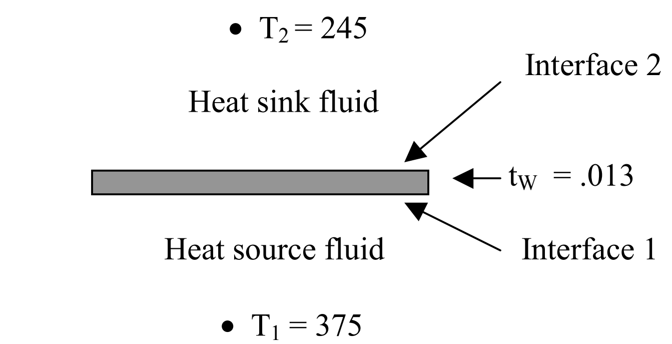
- The wall heat flux in Figure (7-4) is 585 Btu/hrft^2.
- The wall temperature at Interface 1 is 205 F.
- The wall temperature at Interface 2 is 204 F.

7.12 Heat transfer behavior methodology—Problem 7.12

Problem 7.12 differs from Problems 7.10 and 7.11 in that one of the boundary layers exhibits *highly* nonlinear behavior. In spite of the non-linearity, the solution is simple and direct using behavior methodology. (In the next chapter, this problem is to be solved using coefficient methodology. Note that the solution using resistance methodology must be indirect, and is *much* more difficult.)

Problem statement

What is the heat flux in the Figure (7-5) configuration? What is the wall temperature at each interface?



<u>Figure 7-5</u> Heat transfer configuration, Problem 7.12

Given

- Equipment drawings
- Identity of source fluid, sink fluid, and wall material
- Flow rate of source fluid and sink fluid
- The heat transfer coefficient correlation for boundary layer 1 is Eq. (7-41).

$$N_{Nu} = .023\ N_{Re}^{.8}\ N_{Pr}^{.4} \qquad (7\text{-}41)$$

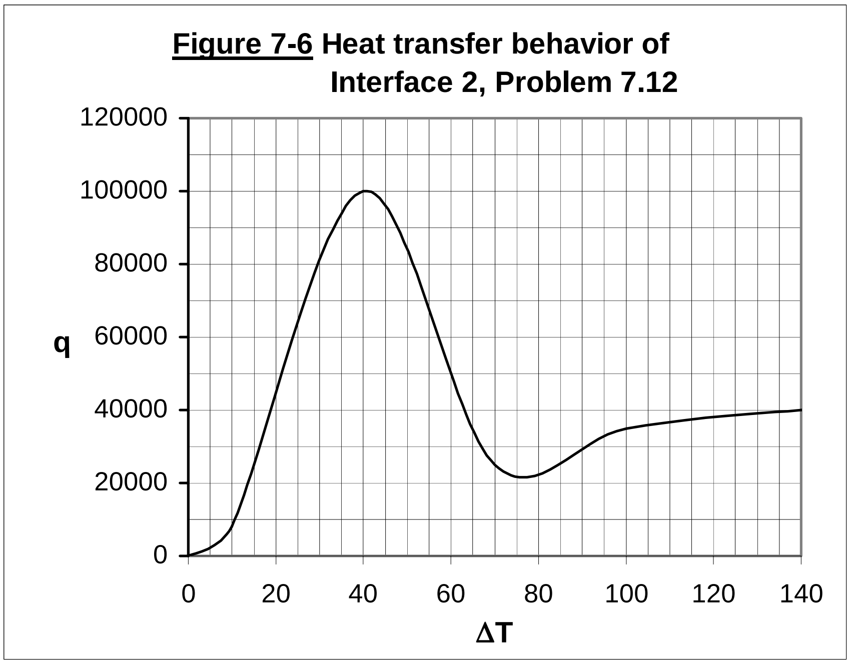
Problem 7.12 cont.

- The heat transfer behavior of boundary layer 2 is described in Figure (7-6).

- The thermal conductivity of the wall is

$$k_W = 110 \text{ Btu/hrftF} \qquad (7\text{-}42)$$

(Note that Eqs. (7-41) and (7-42) are from the literature of conventional heat transfer, and must be converted to the form required in the new heat transfer.)



Analysis

- Transform Eq. (7-41) from a heat transfer coefficient correlation to a heat transfer behavior correlation by substituting (q/ΔT) for *h*, K for *k*, and separating q and ΔT. The result is Eq. (7-43).

$$q = .023\ (K/D)\ N_{Re}^{.8}\ N_{Pr}^{.4}\ \Delta T \qquad (7\text{-}43)$$

Problem 7.12 cont.

- Obtain a specific $q\{\Delta T\}$ equation for boundary layer 1 by evaluating Eq. (7-43) at the given conditions. Assume that evaluation of Eq. (7-43) results in Eq. (7-44).

$$q_1 = 775\ \Delta T_1 \tag{7-44}$$

- Obtain a specific $q\{\Delta T\}$ equation for the wall by determining K_W from k_W in the manner described in Section 7.3, and substituting in Eq. (7-13).

$$q_W = (K_W/t_W)\ \Delta T_W = (110/.013)\ \Delta T_W = 8460\ \Delta T_W \tag{7-45}$$

- Calculate coordinates of $q\{\Delta T_{TOTAL}\}$:
  - o Inspect Figure (7-5) and note that

$$\Delta T_{TOTAL} = T_1 - T_2 = \Delta T_1 + \Delta T_W + \Delta T_2 \tag{7-46}$$

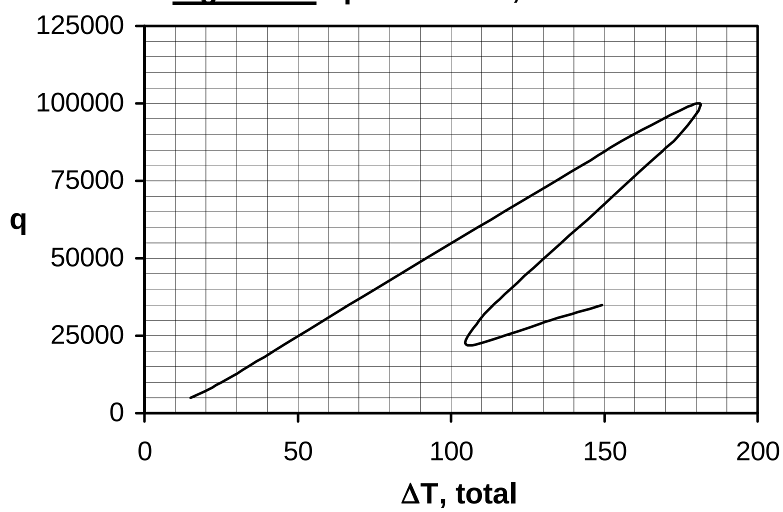
$$q_1 = q_W = q_2 = q \tag{7-47}$$

  - o Select $(q_2, \Delta T_2)$ coordinates from Figure (7-6).
  - o Calculate $\Delta T_1\{q_2\}$ using Eqs. (7-44) and (7-47).
  - o Calculate $\Delta T_W\{q_2\}$ using Eq. (7-45) and (7-47).
  - o Calculate $\Delta T_{TOTAL}\{q\}$ using Eq. (7-46).
  - o The calculated $(q, \Delta T_{TOTAL})$ coordinates are in Table (7-1).
- Plot $(q, \Delta T_{TOTAL})$ coordinates from Table (7-1) in Figure (7-7).

$q_{2 \text{ or }} q$	ΔT_2	ΔT_1	ΔT_W	ΔT_{TOTAL}
5000	8	6	1	15
10000	11	13	1	25
20000	14	26	2	42
40000	19	52	5	76
60000	24	77	7	108
80000	30	103	9	142
90000	33	116	11	160
100000	40	129	12	181
90000	48	116	11	175
80000	51	103	9	164
60000	57	77	7	141
40000	63	52	5	120
30000	67	39	4	110
22000	75	28	3	106
30000	90	39	4	133
35000	100	45	4	149

Table 7-1 Calculation of (q, ΔT_{TOTAL}) coordinates, Problem 7.12

Figure 7-7 q vs total ΔT, Problem 7.12



<u>Problem 7.12 cont.</u>

- Figure (7-7) indicates that $q\{\Delta T_{TOTAL} = 130\}$= 29000, or 50000, or 72000. The information given is not sufficient to uniquely determine $q\{\Delta T_{TOTAL} = 130\}$.

- Eqs. (7-48) and (7-49) are obtained by inspection of Figure (7-5) .

$$T_{W1} = T_1 - \Delta T_1 = 375 - \Delta T_1 \qquad (7\text{-}48)$$

$$T_{W2} = T_{W1} - \Delta T_W \qquad (7\text{-}49)$$

- The wall temperature at Interface 1 is obtained from Eq. (7-48), using Eq. (7-44) and the q values from Figure (7-7).. The results are T_{W1} = 338, or 310, or 282 at q = 29000, or 50000, or 72000.

- The wall temperature at Interface 2 is obtained from Eq. (7-49), using Eq. (7-45) and the q values from Figure (7-7). The results are T_{W2} = 335 or 304 or 273 at q = 29000 or 50000 or 72000.

Solution

The information given is not sufficient to determine a unique solution for $q\{\Delta T_{TOTAL} = 130\}$. The three possible solutions are:

q	T_{W1}	T_{W2}
29000	338	335
50000	310	304
72000	282	273

7.13 Conclusions

Proportional and nonlinear heat transfer problems are solved in a simple and direct manner using behavior methodology.

Chapter 8

The heat transfer coefficient form of the problems in Chapter 7

8 Introduction

In Chapter 7, heat transfer problems are stated and solved using heat transfer *behavior* methodology. In this chapter, the problems in Chapter 7 are restated (but not solved) using heat transfer *coefficient* methodology

The reader is encouraged to solve the problems in this chapter using heat transfer coefficient methodology. By comparing his/her coefficient solutions with the behavior solutions presented in Chapter 7, the reader will gain a first hand appreciation of the simplicity that results from using behavior methodology rather than coefficient methodology.

The problems, figures, and equations in this chapter are *identical* to those in Chapter 7. They differ only in form. The behavior form is used throughout Chapter 7, the coefficient form is used throughout this chapter.

Corresponding problems, figures, and equations in this chapter have the same identifying numbers used in Chapter 7, except that "C" is added to the identifying numbers in this chapter (to denote coefficient form). For example, Problem (7.9C) is the coefficient form of Problem (7.9) in Chapter 7. Eq. (7-20C) is the coefficient form of Eq. (7-20) in Chapter 7.

8.1 Problem solutions based on *h*

The problems in this chapter demonstrate that problem solutions based on h are simple and direct if the phenomena involved are strictly proportional, but are generally *not* simple or direct if the phenomena involved are nonlinear. h is the symbol for $q/\Delta T$, and therefore h makes it necessary to solve problems with q and ΔT combined. Nonlinear problems can be solved in a direct manner if q and ΔT are separated—as in Ch. 7.

8.2 The coefficient form of the problems in Chapter 7

Problem 7.10C

Problem statement

What is the heat flux in the Figure (7-3C) configuration? What is the wall temperature at each interface?

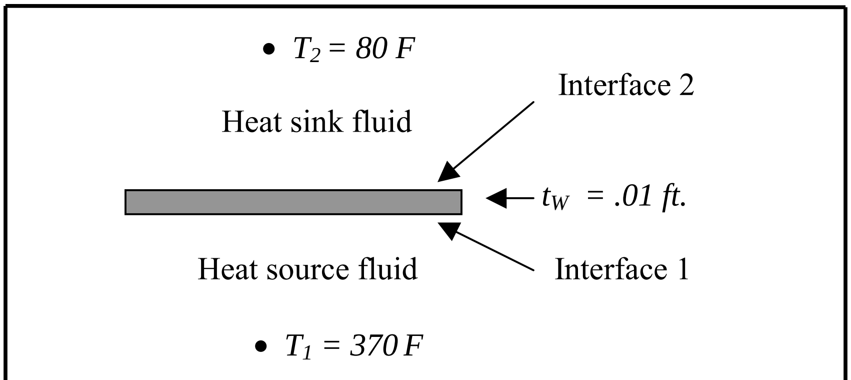


Figure 7-3C **Heat transfer configuration, Problem 7.10C**

Given

- Equipment drawings
- Identity of source fluid, sink fluid, and wall material
- Flow rate of source fluid and sink fluid
- The heat transfer coefficients for boundary layer 1 and boundary layer 2 are described by Eq. (7-16C), obtained from conventional engineering literature literature.

$$N_{Nu} = .023\ N_{Re}^{.8}\ N_{Pr}^{.4} \qquad (7\text{-}16C)$$

Problem 7.10C cont.

- The thermal conductivity of the wall is described by Eq. (7-17C), obtained from conventional engineering literature.

$$k_W = 14.5 \text{ Btu/hrft} \tag{7-17C}$$

- The fluid and geometry parameters in Eq. (7-16C) were evaluated using given information and the literature. Eqs. (7-19C) and (7-20C) resulted.

$$h_1 = 158 \text{ Btu/hrft}^2\text{F} \tag{7-19C}$$

$$h_2 = 85 \text{ Btu/hrft}^2\text{F} \tag{7-20C}$$

Analysis and solution

(To be determined by the reader.)

Problem 7.11C

Problem statement

What is the heat flux in the Figure (7-4C) configuration? What is the wall temperature at each interface?

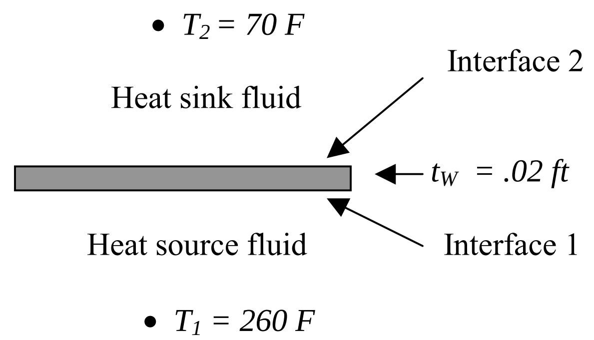


Figure 7-4C Heat transfer configuration, Problem 7.11C

Given

- Equipment drawings
- Identity of source fluid, sink fluid, and wall material
- Flow rate of source fluid and sink fluid.
- The heat transfer coefficient at boundary layer 1 is described by Eq. (7-28C). The heat transfer coefficient at boundary layer2 is described by Eq. (7-29C).

$$N_{Nu1} = 0.15\ (N_{Gr1}\ N_{Pr1})^{.33} \qquad (7\text{-}28C)$$

$$N_{Nu2} = 0.47\ (N_{Gr2}\ N_{Pr2})^{.20} \qquad (7\text{-}29C)$$

- The fluid and geometry parameters in Eqs. (7-28C) and (7-29C) were evaluated using given information and the literature. The following values were obtained:

Problem 7.11C cont.

	(k/L) Btu/hrft^{2}F	***($g\beta L^3/\nu^2$)*** F^{-1}	N_{Pr}
Boundary layer 1	.23	360000	1.7
Boundary layer 2	.31	250000	2.4

- The thermal conductivity of the wall is given by Eq. (7-30C).

k_W = 8.6 Btu/hrftF (7-30C)

Analysis and solution
(To be determined by the reader.)

Problem 7.12C

Problem statement

What is the heat flux in the Figure (7-5C) configuration? What is the wall temperature at each interface?

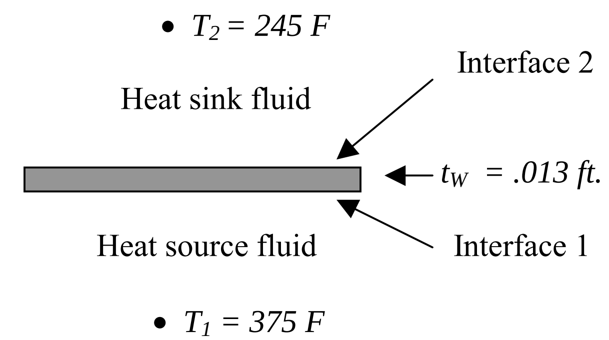


Figure 7-5C Heat transfer configuration, Problem 7.12C

Given

- Equipment drawings
- Identity of source fluid, sink fluid, and wall material
- Flow rate of source fluid and sink fluid.
- The heat transfer coefficient at boundary layer 1 is described by Eq. (7-41C).

$$N_{Nu} = .023\, N_{Re}^{.8}\, N_{Pr}^{.4} \qquad (7\text{-}41C)$$

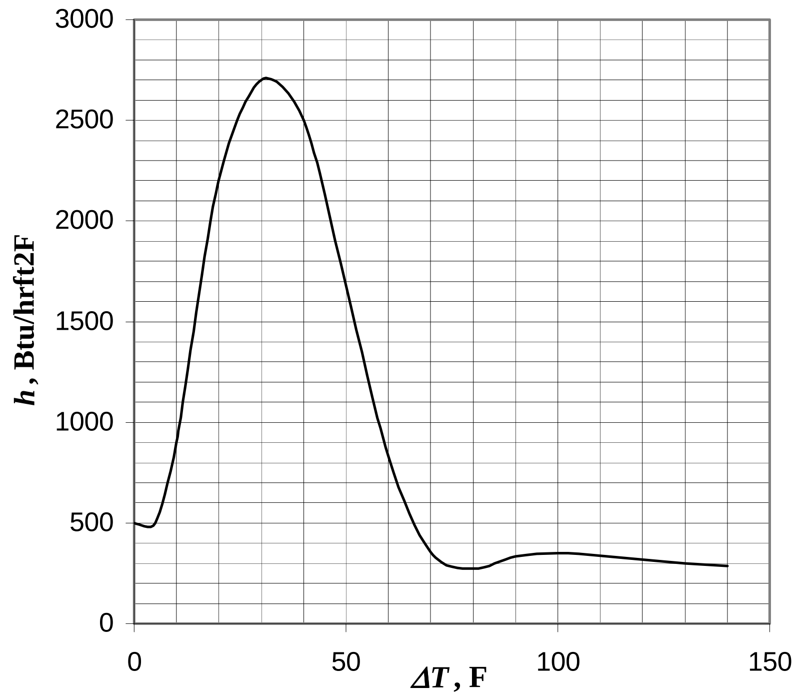
- The fluid and geometry parameters in Eq. (7-41C) were evaluated, and Eq. (7-44C) resulted.

$$h_1 = 775 \text{ Btu/hrft}^2\text{F} \qquad (7\text{-}44C)$$

Problem 7.12C cont.

- The heat transfer coefficient at boundary layer 2 is described by Figure (7-6C).

Figure 7-6C Heat transfer coefficient at Interface 2, Problem 7.12C



- The thermal conductivity of the wall is given by Eq. (7-42C).

$$k_W = 110 \text{ Btu/hrftF} \quad (7\text{-}42C)$$

Analysis and Solution

(To be determined by the reader.)

Chapter 9A

Why convective heat transfer behavior q{ΔT} should replace heat transfer coefficient *q/ΔT*

9A Introduction

This chapter addresses the question

> *Should convective heat transfer behavior q{ΔT} replace heat transfer coefficient q/ΔT?*

The question is answered in two ways:

- In a general way by appraising and comparing concepts and methodologies.

- In a specific way by comparing the behavior analyses in Chapter 7 with the coefficient analyses of the same problems in Chapter 8.

The answers strongly support the conclusion that convective heat transfer behavior should replace heat transfer coefficient.

9A.1 The de facto view of heat transfer coefficient

For almost 200 years, heat transfer coefficients have been used to describe, analyze, and predict heat transfer phenomena. Heat transfer coefficients are so fundamental and so important in heat transfer that they are generally discussed in the first lecture of the first undergraduate course in heat transfer.

The end result of this long history is that heat transfer coefficient has come to be viewed as a fundamental parameter of Nature—a parameter as real as electromotive force or temperature—a parameter whose existence *cannot* be denied.

Based on the de facto view of heat transfer coefficient, it is preposterous to question its value.

9A.2 The original view of heat transfer coefficient

Fourier (1822) performed numerous heat transfer experiments, from which he concluded that convective heat flux is *generally* proportional to the temperature difference between an object and the surrounding fluid:

$$q_{conv} \propto \Delta T \qquad \text{(9A-1)}$$

Fourier converted this empirical and global expression of proportionality into an equation by introducing an arbitrary constant, to which he assigned the name "coefficient" and the symbol h. The result was Eq. (9A-2).

$$q_{conv} = h\, \Delta T \qquad \text{(9A-2)}$$

Inexplicably, American texts on conventional heat transfer generally refer to Eq. (9A-2) as "Newton's law of cooling", and credit Newton (1701) with the heat transfer coefficient concept and Eq. (9A-2). However, they were in fact conceived by Fourier (1822). (See Adiutori, 1974 and 1990.)

In Fourier's view, Eq. (9A-2):

- Is a *global* description of the relationship between q_{conv} and ΔT.
- States that q_{conv} is proportional to ΔT.
- Assigns the symbol h to the constant of proportionality that relates q_{conv} and ΔT.

Fourier also pioneered the view that scientific rigor requires equations to be dimensionally homogeneous—ie all terms in an equation must be of the same dimension—ie the equal sign indicates numerical equality *and* dimensional identity. In order to satisfy his view of homogeneity, Fourier made Eq. (9A-2) homogeneous by arbitrarily assigning the necessary dimensions to the proportionality constant h.

Fourier's view of heat transfer coefficient and Eq. (9A-2) prevailed for many decades, and his view of dimensional homogeneity still prevails in conventional engineering.

9A.3 The need to alter Fourier's view of Eq. (9A-2) and *h*

Decades after Fourier, it was recognized that Eq. (9A-2) does *not* globally describe convective heat transfer phenomena. q is *not* always proportional to ΔT, and therefore the ratio $q/\Delta T$ oftentimes depends on ΔT—ie the value of h often depends on the value of ΔT.

For example, q is not proportional to ΔT in free convection, condensation, or boiling. Therefore it became necessary either to alter Fourier's view of Eq. (9A-2) and h, or to abandon them.

In conventional engineering, Fourier's view of Eq. (9A-2) and h has been altered. In the new engineering, Eq. (9A-2) and h have been abandoned.

9A.4 The conventional engineering view of Eq. (9A-2) and *h*

Texts on conventional heat transfer generally describe heat transfer coefficient in words similar to the following:

> *Newton's law of cooling, Eq. (9A-2), is not a phenomenological description of behavior. It is the defining equation for h.*

Notice that the conventional heat transfer view differs from Fourier's view in the following ways:

- Eq. (9A-2) is *not* a description of heat transfer behavior. It is a definition of h.

- Eq. (9A-2) does *not* state that h is the proportionality constant between q and ΔT. It states that h is a symbol for the ratio $q/\Delta T$.

- Eq. (9A-2) does *not* state that the ratio $q/\Delta T$ (symbol h) is constant—ie is independent of ΔT. It may be constant, or it may be a variable function of ΔT.

- In general, the ratio $q/\Delta T$ (symbol h) must be treated as a *variable*.

9A.5 A more appropriate form of Eq. (9A-2)

Eq. (9A-2) is misleading because it is written in a form that suggests it describes the relationship between q and ΔT, when in fact it does not. Since Eq. (9A-2) is "the defining equation for h", it would be more appropriate to write it in the form of a definition, as in Definition (9A-3).

$$h \equiv \text{heat transfer coefficient} \equiv q/\Delta T \qquad (9A\text{-}3)$$

Notice that Definition (9A-3) clearly and correctly states the conventional view:

- The ratio $q/\Delta T$ is assigned the name heat transfer "coefficient" and the symbol h.
- The definition makes no statement about the relationship between q and ΔT. The relationship may be proportional, or linear, or nonlinear.
- The ratio $q/\Delta T$ (symbol h) may be constant or variable, depending on the relationship between q and ΔT.
- The ratio $q/\Delta T$ must in general be treated as a variable function of ΔT.

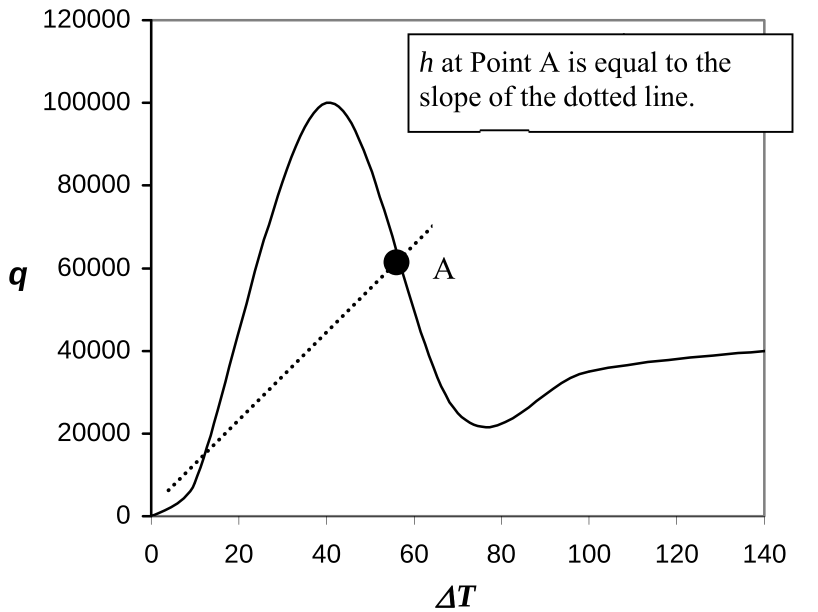
Also notice that Definition (9A-3) correctly indicates that h can be experimentally determined *only* by individually measuring q and ΔT, and then using the data to calculate values of $q/\Delta T$.

9A.6 The graphical analog of heat transfer coefficient

Without exception, texts on conventional heat transfer do *not* present the graphical analog of h. Presumably, the reason for this error of omission is that h is viewed as a fundamental parameter, and fundamental parameters do not have graphical analogs. For example, temperature has no graphical analog.

The graphical analog of h is shown in Figure (9A-1). The analog is verbally described by the following:

> On a *linear* graph with the origin at (0,0), the graphical analog of h at a point on $q\{\Delta T\}$ is the slope of a line drawn from the origin to the point on $q\{\Delta T\}$.



<u>Figure 9A-1</u> Graphical analog of *h* at Point A.

Note the following in Figure (9A-1):

- *h* at any point on the curved line is *q{ΔT}/ΔT*. The slope of the dotted line is *q{ΔT}/ΔT*. Therefore the slope of the dotted line equals the value of *h* at Point A.

- The value of *q{ΔT}/ΔT* (symbol *h*) reveals little about *q{ΔT}*. It reveals only that, *somewhere in space*, *q{ΔT}* makes at least one intersection with a line whose slope is the known value of *q{ΔT}/ΔT*.

- The slope of the dotted line and *h* are *always* positive, but the slope of *q{ΔT}* may be positive or negative. In other words, Eq. (9A-2) and *h* indicate that, at *any* point on *q{ΔT}*, an incremental increase in *ΔT* would result in an *in*creased value of *q*. But note that at Point A and throughout the negative slope region in Figure (9A-1), an incremental *in*crease in *ΔT* would result in a *de*creased value of *q*.

- If Figure (9A-1) were a *y{x}* chart in a mathematics class, there would be *no* discussion of a line drawn from the origin to a point on *y{x}*. Mathematics has no use for such a line. Its slope is *y/x*, a term that combines the variables *x* and *y*, and thereby complicates the solution of mathematical problems.

- If Figure (9A-1) were a *y{x}* chart in a mathematics class, a tangent line would be drawn at Point A, and it would be noted that the slope of the line is the first derivative of *y{x}*. Note that conventional heat transfer has *no use* for the slope of a tangent line at a point on *q{ΔT}*—*no use* for *dq/dΔT*.

- If Figure (9A-1) were an s{t} chart, the slope of the dotted line would be (s/t), the time-averaged velocity. It would be noted that (s/t) is not used in motion analysis, although (s/t) is sometimes a desired result. It would also be noted that instantaneous velocity at time t is the slope of a tangent line at time t on s{t}.

- The dotted line provides a satisfactory description of *q{ΔT} only* if *q* is proportional to *ΔT*. In that event,

    - The dotted line would fall on the *q{ΔT}* line.

    - The dotted line would correctly and completely describe *q{ΔT}*.

    - *h* and *dq/dΔT* would be equal.

In summary, the graphical analog of *h* at Point A on *q{ΔT}* is the slope of the dotted line in Figure (9A-1). The slope of a line from the origin to a point on a function is not used in mathematics or motion analysis because it reveals very little about the function. Yet it is the basis for heat transfer analysis in conventional engineering.

9A.7 Why *q/ΔT* (symbol *h*) is undesirable

q/ΔT (symbol *h*) is undesirable because it combines *q* and *ΔT*, making it necessary to solve heat transfer problems with the variables combined.

If a problem concerns proportional phenomena, it can be solved in a simple and direct manner whether the variables are combined or separated. If a problem concerns nonlinear phenomena, it can be solved in a direct manner if the variables are separated, but must generally be solved in an indirect manner if the variables are combined.

Because indirect solutions are more difficult than direct solutions, combining the variables complicates the solution of nonlinear problems.

9A.8 The relationship between behavior methodology and coefficient methodology

Heat transfer behavior methodology and heat transfer coefficient methodology are *identica*l. They differ only in form.

- In behavior methodology, q and ΔT are *separate* and *explicit.*
- In resistance methodology, q and ΔT are *combined* and *implicit* in h, the symbol for $q/\Delta T$.

For example, in coefficient methodology, the heat flux between two fluids separated by a wall is determined using Eqs. (9A-4) and (9A-5).

$$U \equiv q/\Delta T_{TOTAL} \tag{9A-4}$$

$$U = (1/h_1 + t/k_W + 1/h_2)^{-1} \tag{9A-5}$$

Note that q and ΔT are combined and implicit in U, k_W, h_1, and h_2.

In behavior methodology, the heat flux between two fluids separated by a wall is determined from Eq (9A-6) in the manner illustrated in Chapter 7.

$$\Delta T_{TOTAL} = \Delta T_1 + \Delta T_W + \Delta T_2 \tag{9A-6}$$

It is important to note that Eq. (9A-6) is *identical* to Eq. (9A-5). The equations make *exactly* the same statement, but in different forms. Equation (9A-5) is in the form of combined variables. Eq. (9A-6) is in the form of separated variables.

Eqs. (9A-5) and (9A-6) appear to be so different that it is necessary to prove that they are identical. The proof is obtained by showing that Eq. (9A-6) results when q and ΔT in Eq. (9A-5) are separated.

Separation is accomplished by first substituting the following in Eq. (9A-5): $q/\Delta T_{TOTAL}$ for U, $q/\Delta T_1$ for h_1, $qt/\Delta T_W$ for k_W, and $q/\Delta T_2$ for h_2, resulting in Eq. (9A-7):

$$(q/\Delta T_{TOTAL}) = (\Delta T_1/q + \Delta T_W/q + \Delta T_2/q)^{-1} \tag{9A-7}$$

$$\therefore (q/\Delta T_{TOTAL}) = q/(\Delta T_1 + \Delta T_W + \Delta T_2) \qquad (9A\text{-}8)$$

$$\therefore \Delta T_{TOTAL} = \Delta T_1 + \Delta T_W + \Delta T_2 \qquad (9A\text{-}6)$$

In summary, behavior methodology and coefficient methodology are *identical*. They differ only in form. In behavior methodology, q and ΔT are *separate* and *explicit*. In coefficient methodology, *q* and *ΔT* are *combined* and *implicit* in *h*, the symbol for *q/ΔT*.

9A.9 The principal advantage of behavior methodology

The principal advantage of behavior methodology is that it allows problems to be solved with the variables separated, and this greatly simplifies the solution of nonlinear problems. Chapter 7 illustrates the solution of heat transfer problems using behavior methodology—ie with q and ΔT separate and explicit. The problems demonstrate that:

- Heat transfer coefficients are unnecessary, since the problems are stated and solved without heat transfer coefficients.
- Proportional *and* nonlinear heat transfer problems are solved in a direct manner using behavior methodology.

Chapter 8 presents the Chapter 7 problems in terms of coefficient methodology, and requests that the reader solve the problems using coefficient methodology. The reader is further requested to compare her/his solutions with those presented in Chapter 7. The comparison supports the following conclusions:

- Proportional problems can be solved in a simple and direct manner using either behavior methodology or coefficient methodology.
- Nonlinear problems that must be solved in an indirect manner using coefficient methodology can be solved in a direct and much simpler manner using behavior methodology

9A.10 Other advantages of behavior methodology

Other advantages of behavior methodology relative to coefficient methodology are:

- Behavior methodology is easier to learn because problems are solved using methodology learned in mathematics—ie problems are solved with the variables separated rather than combined.
- Behavior methodology is easier to learn because it obviates the need to learn about coefficients and their application.

- Behavior methodology is more logical. It is more logical to solve problems that involve q and ΔT using q and ΔT instead of *q* and *ΔT and q/ΔT* (symbol *h*).

  In other words, it is more logical to solve problems that involve two variables using two variables rather than three variables where the third variable is the ratio of the other two variables.

9A.11 Conclusions

- Eq. (9A-2) and *q/ΔT* (symbol *h*) should be abandoned.

- Convective heat transfer behavior q{ΔT} should be used to describe, analyze, and predict convective heat transfer phenomena.

- Ratios of primary parameters should generally be abandoned in favor of behavior methodology.

Chapter 9B

Why conductive heat transfer behavior q{dT/dx} should replace thermal conductivity *q/(dT/dx)*

9B Introduction

This chapter addresses the question

> *Should conductive thermal behavior q{dT/dx} replace thermal conductivity q/(dT/dx)?*

The answer is obtained by noting that, since conductive heat transfer globally exhibits proportional behavior, there is currently no practical difference between conductive thermal behavior and thermal conductivity. Therefore there is no pressing need to replace thermal conductivity at this time.

However, in the interest of a consistent heat transfer science, and because materials invented in the future may exhibit conductive heat transfer behavior that is nonlinear, conductive thermal behavior q{dT/dx} should now replace thermal conductivity *q/(dT/dx)*.

9B.1 The original and still current view of conductivity

Fourier (1822) performed numerous heat transfer experiments, from which he concluded that conductive heat flux is generally proportional to temperature gradient:

$$q_{cond} \propto dT/dx \qquad (9B\text{-}1)$$

Fourier converted this empirical and global expression of proportionality to an equation by introducing a constant. He arbitrarily assigned the constant the name "conductivity", the symbol *k*, and the dimensions that would make the equation homogeneous. He noted that the constant is a weak function of temperature. The end result was Eq. (9B-2):

$$q_{cond} = k\{T\}(dT/dx) \qquad (9B\text{-}2)$$

In Fourier's view, Eq. (9B-2):

- States that q_{cond} is *globally* proportional to *dT/dx*.
- States that *k* is the proportionality constant between q_{cond} and *dT/dx*.
- Is dimensionally homogeneous.

Fourier's view is still applicable, and it is the view held in conventional engineering.

9B.2 Conductive heat transfer behavior

Eq. (9B-3) is the generic equation for conductive heat transfer behavior in the new engineering.

$$q_{cond} = f\{T, dT/dx\} \qquad (9B\text{-}3)$$

Because all practical materials exhibit proportional conductive behavior, Eq. (9B-4) is now also generic.

$$q_{cond} = K\{T\}(dT/dx) \qquad (9B\text{-}4)$$

Recall from Chapter 7 that K{T} is the proportionality constant between q_{COND} and dT/dx.

9B.3 The impact of nonlinear conductive behavior

Because all practical materials exhibit proportional conductive behavior, Eqs. (9B-2) through (9B-4) currently describe conductive behavior in a global way. However, if at some future time materials are invented that exhibit nonlinear conductive behavior, then:

- Eq. (9B-2) will still apply globally, but it will no longer be possible to view *k{T}* generically as the proportionality constant between *q* and *dT/dx*. It will have to be viewed generically as the ratio $q_{cond}/(dT/dx)$.

  (Recall that after it was recognized that certain convective heat transfer phenomena exhibit nonlinear behavior, *h* could no longer be viewed generically as the proportionality constant between *q* and *ΔT*. It had to be viewed generically as the variable ratio *q/ΔT*.)

- Eq. (9B-3) will still apply globally, but Eq. (9B-4) will apply only to those materials that exhibit proportional conductive behavior.

9B.4 The need to replace thermal conductivity with conductive thermal behavior

Because all practical materials exhibit proportional conductive behavior, there is *now* no practical difference between *k{T}* and K{T}. Therefore there is no pressing need to replace thermal conductivity with conductive thermal behavior *at this time*.

However, there are two reasons conductive thermal behavior q{dT/dx} should *now* replace thermal conductivity *q/(dT/dx)*:

- Some time in the future, materials may be invented that exhibit nonlinear conductive behavior. When that happens, there *will be* a pressing need to replace thermal conductivity with conductive thermal behavior, just as there is now a pressing need to abandon heat transfer coefficient, and replace it with convective thermal behavior.

- Retaining thermal conductivity while abandoning heat transfer coefficient would result in a piecemeal and aesthetically unsatisfactory heat transfer science.

It therefore seems advisable to now replace thermal conductivity with conductive thermal behavior, even though there is not now a pressing need to do so.

9B.5 Conclusions

- Eq. (9B-2) and *q/(dT/dx)* (symbol *k*) should be abandoned.

- Conductive heat transfer behavior q{dT/dx} should be used to describe, analyze, and predict conductive heat transfer phenomena.

- All ratios of primary parameters should be abandoned in favor of behavior methodology, even if the primary parameters are currently globally proportional to each other.

Chapter 10

Stability of heat transfer systems, and summary

10 Introduction

Instability in heat transfer systems is a practical problem only if the system includes a heat transfer element for which ($dq/d\Delta T$) is negative over some region of the system operating range. This type of behavior is commonly exhibited only by boiling boundary layers, and thus heat transfer system instability is a practical problem only in boiling systems. The problem is particularly severe in boiling liquid metal systems.

In this chapter, the stability of heat transfer systems and the performance of unstable heat transfer systems are analyzed using behavior methodology. The analyses can also be performed using coefficient methodology, but the extreme nonlinearity involved causes stability analyses based on coefficient methodology to be so difficult that there is little point in considering them.

The problems in this chapter illustrate that behavior methodology deals with heat transfer systems in a simple and direct manner, even if they contain elements that exhibit the highly nonlinear behavior that can result in unstable behavior.

10.1 The stability question

The stability analyses in this chapter answer the question:

> *If a system is initially at a potential operating point, will the system resist a very small perturbation, and return to the potential operating point?*

If the answer is "no", the system is "unstable" at the potential operating point—ie it will *not* operate in a steady-state manner at that point. However, it may be quite stable at other potential operating points.

If the answer is "yes", the system is *conditionally* "stable" at the potential operating point—ie it will operate in a steady-state manner at that point *provided* all perturbations are small. The system is *conditionally* stable because, even though it is stable with respect to *small* perturbations, it may be unstable with respect to *large* perturbations.

10.2 The effect of instability

If a system is initially at an unstable operating point and is left alone, the system will tend to leave the unstable point. One of the following will result:

- Hysteresis.

- Undamped oscillation.

The heat transfer behavior of the components determines whether instability results in hysteresis or undamped oscillation.

10.3 Uncoupling the system in order to analyze stability

In system analysis, it is often convenient to:

- Uncouple the system—ie divide it into subsystems.

- Analytically determine the behavior of each subsystem.

- Analytically determine the system performance that would result from coupling the subsystems.

The above method is used here to analyze the stability of heat transfer systems. The systems analyzed contain a heat source fluid, a heat sink fluid, and a wall that separates the two fluids. One of the fluid boundary layers exhibits highly nonlinear heat transfer behavior that includes a region in which $(dq/d\Delta T)$ is negative. The method includes the following steps:

- Uncouple the system at the wall surface that adjoins the nonlinear boundary layer. One subsystem includes the highly nonlinear boundary layer and its fluid. The other subsystem includes the wall, the other boundary layer, and its fluid.

- Note that the wall surface that adjoins the nonlinear boundary layer is the interface between the two subsystems, and is referred to by the subscript INTERFACE.

- Subscript "IN" refers to the subsystem that includes the heat source fluid. IN is used to indicate that heat flows from the source fluid INto the interface.

- Subscript "OUT" refers to the subsystem that includes the heat sink fluid. OUT is used to indicate that heat flows OUT of the interface and into the sink fluid.

- Determine $q_{IN}\{T_{INTERFACE}\}$.

- Determine $q_{OUT}\{T_{INTERFACE}\}$.

- Plot $q_{IN}\{T_{INTERFACE}\}$ and $q_{OUT}\{T_{INTERFACE}\}$ together on the same graph.

- Note that intersections of $q_{IN}\{T_{INTERFACE}\}$ and $q_{OUT}\{T_{INTERFACE}\}$ are potential operating points.

- Use Criterion (10-1) to appraise the stability of the system at potential operating points.

10.4 The criterion for heat transfer system instability

Criterion (10-1) is the criterion for heat transfer system instability:

$$dq_{IN}/dT_{INTERFACE} \geq_U dq_{OUT}/dT_{INTERFACE} \qquad (10\text{-}1)$$

The criterion states:

> If a heat source subsystem is coupled to a heat sink subsystem, the resultant system will be *unstable* at a potential operating point if $dq_{IN}/dT_{INTERFACE}$ is greater than or equal to $dq_{OUT}/dT_{INTERFACE}$. (The $\geq_U$ symbolism indicates "unstable if satisfied".)

The criterion describes stability with regard to *very small* perturbations. Therefore:

- If the criterion is satisfied at a potential operating point, the system is unstable at that potential operating point with regard to small perturbations.

- If the criterion is *not* satisfied at a potential operating point, the system is stable at that potential operating point with respect to very small perturbations. However, it may or may not be stable with respect to large perturbations.

In this chapter, a system is described as "stable" at a potential operating point if Criterion (10-1) is not satisfied. However, it must be recognized that "stable" is used as a shorthand expression for "stable with regard to very small perturbations".

The system design objective is generally "stable with respect to perturbations inherent in the system". Fortunately, background perturbations in real systems are generally quite small. Thus there is usually little practical difference between "stable with respect to small perturbations", and "stable with respect to perturbations inherent in the system".

10.5 Verifying Criterion (10-1)

Criterion (10-1) can be verified by showing that, if a heat source/sink system is initially at a potential operating point, a small perturbation will tend to grow if Criterion (10-1) is satisfied.

Figure (10-1) describes the heat transfer behavior of coupled subsystems. The intersection in Figure (10-1) is a potential operating point. The stability at the intersection can be appraised in the following way:

- Assume that the system described in Figure (10-1) is initially operating at the intersection.

- Suddenly the system experiences a very small, positive perturbation in $T_{INTERFACE}$.

- The positive perturbation causes $q_{IN}\{T_{INTERFACE}\}$ to be greater than $q_{OUT}\{T_{INTERFACE}\}$. In other words, the heat flow into the interface exceeds the heat flow out of the interface.
- Because the heat flow into the interface is greater than the heat flow out, the temperature of the interface *increases* with time.

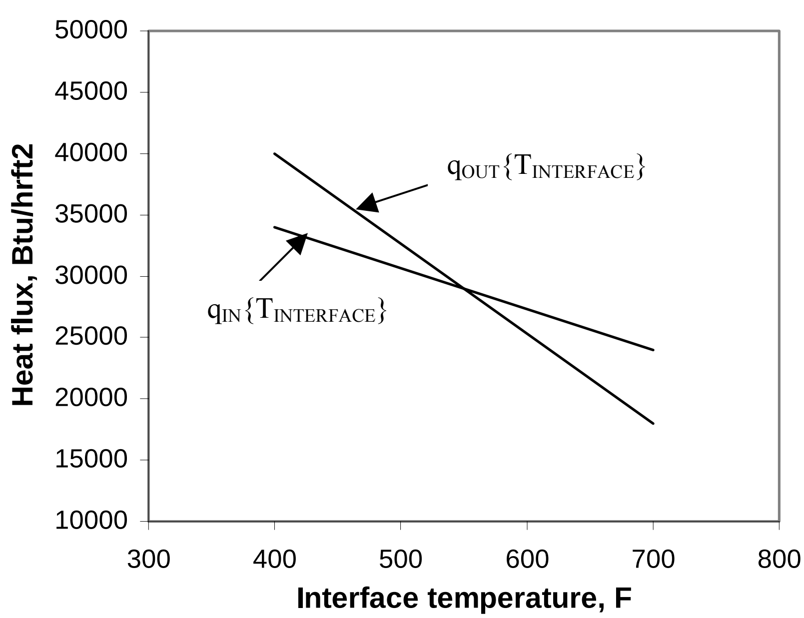
- An increasing $T_{INTERFACE}$ indicates that the system is not returning to the potential operating point. Therefore the intersection in Figure (10-1) is an unstable operating point.

- To determine whether Criterion (10-1) also indicates instability, note that the slope of $q_{IN}\{T_{INTERFACE}\}$ is greater than the slope of $q_{OUT}\{T_{INTERFACE}\}$ Since this satisfies Criterion (10-1), the criterion indicates instability. (Note that, since both slopes are negative, the *greater* slope is *less* steep.)

- Since the above analysis and Criterion (10-1) are in agreement, the analysis validates Criterion (10-1).

- Notice that, if $q_{IN}\{T_{INTERFACE}\}$ and $q_{OUT}\{T_{INTERFACE}\}$ were interchanged in Figure (10-1), a positive perturbation would cause $T_{INTERFACE}$ to decrease with time. Therefore Criterion (10-1) would not be satisfied, and the system would be stable at the intersection.

Figure 10-1 Potential operating point



10.6 Problem 10.6—Hysteresis in heat transfer systems

Problem 10.6 demonstrates:

- How to analyze a heat transfer system for instability.
- How to determine the effect of heat transfer system instability on system performance.

Problem statement

Describe the performance of the heat transfer system in Figure (10-2) over the source fluid temperature range of 250 to 475 F. (Note that the system may be envisioned as a differential element in a vented pool boiler.)

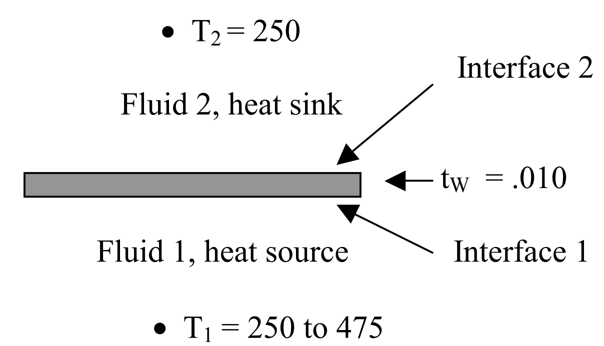


Figure 10-2 Heat transfer configuration, Problem 10.6

Given

- Equipment drawings.
- Identity of heat source fluid, heat sink fluid, and wall material.
- Flow rate of source fluid and sink fluid.
- The heat transfer behavior of boundary layer 1 is described by Eq. (10-2).

$$q_1 = .023\ (K/D)\ N_{Re}^{.8}\ N_{Pr}^{.4}\ \Delta T_1 \qquad (10\text{-}2)$$

Problem 10.6 cont.

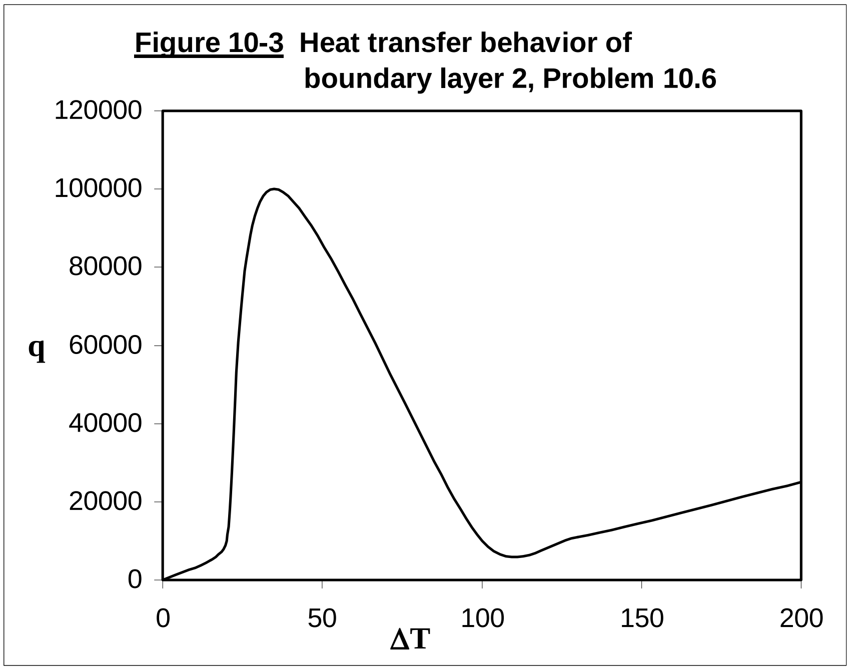
- Evaluation of Eq. (10-2) gives Eq. (10-3).

$$q_1 = 830\ \Delta T_1 \qquad (10\text{-}3)$$

- The heat transfer behavior of the wall is described by Eq. (10-4).

$$q_W = 113\ \Delta T/t_W \qquad (10\text{-}4)$$

- The behavior of boundary layer 2 is described by Figure (10-3).



Analysis

- Substitute in Eq. (10-4), and obtain Eq. (10-5).

$$q_W = 11300\ \Delta T_W \qquad (10\text{-}5)$$

- Uncouple the system at the wall surface adjacent to Fluid 2, and determine $q_{IN}\{T_{W2}\}$ and $q_{OUT}\{T_{W2}\}$. (T_{W2} is the wall temperature adjacent to Fluid 2.)

Problem 10.6 cont.

- Determine $q_{IN}\{T_{W2}\}$ from inspection of Figure (10-2), and Eqs. (10-3) and (10-5).

$$T_1 - T_{W2} = \Delta T_1 + \Delta T_W = q_1/830 + q_W/11300 \qquad (10\text{-}6)$$

$$q_1 = q_W = q_{IN} = q \qquad (10\text{-}7)$$

$$T_1 - T_{W2} = q/830 + q/11300 = q_{IN}/773 \qquad (10\text{-}8)$$

$$q_{IN} = 773\,(T_1 - T_{W2}) \qquad (10\text{-}9)$$

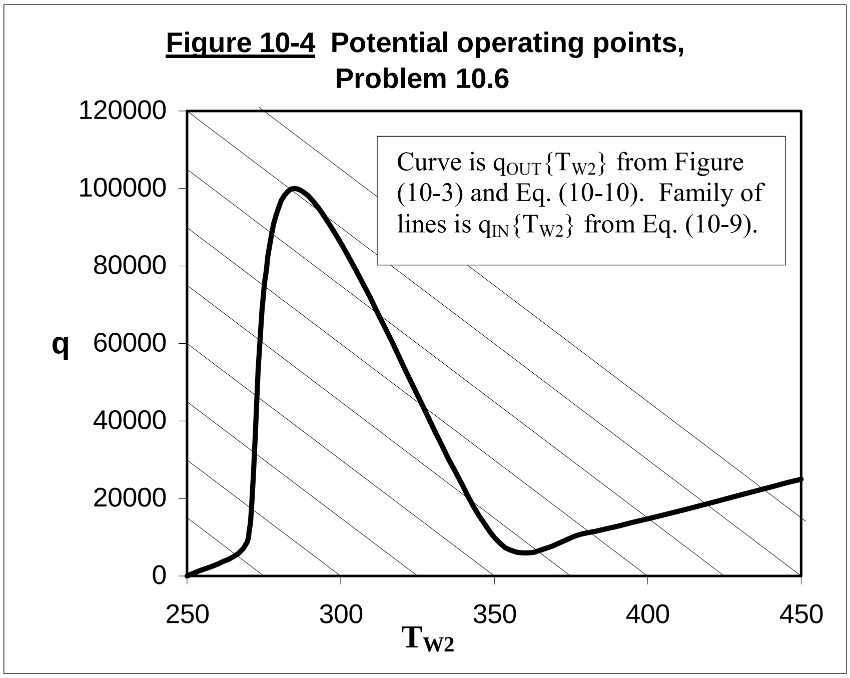
- Note that Figure (10-3) is in the form $q_{OUT}\{\Delta T_{OUT}\}$, and transform the figure to $q_{OUT}\{T_{W2}\}$ using Eq. (10-10). In other words, transform Figure (10-3) to the desired form by adding 250 to the coordinates on the x axis. The result is the curve in Figure (10-4).

$$T_{W2} = T_{SINK} + \Delta T_2 = 250 + \Delta T_2 \qquad (10\text{-}10)$$

- On Figure (10-4), plot Eq. (10-9) for various values of T_1 over the operating range from 250 to 475 F.

- Note that intersections in Figure (10-4) are potential operating points.

Problem 10.6 cont.



Solution

The solution of Problem 10.6 is a q{T_1) chart that covers the source fluid temperature range of 250 to 475 F. The chart is prepared as follows:

- Note from Eq. (10-9) that $T_1 = T_{W2}\{q = 0\}$. Therefore T_1 in Figure (10-4) equals the value of T_{W2} at q = 0.

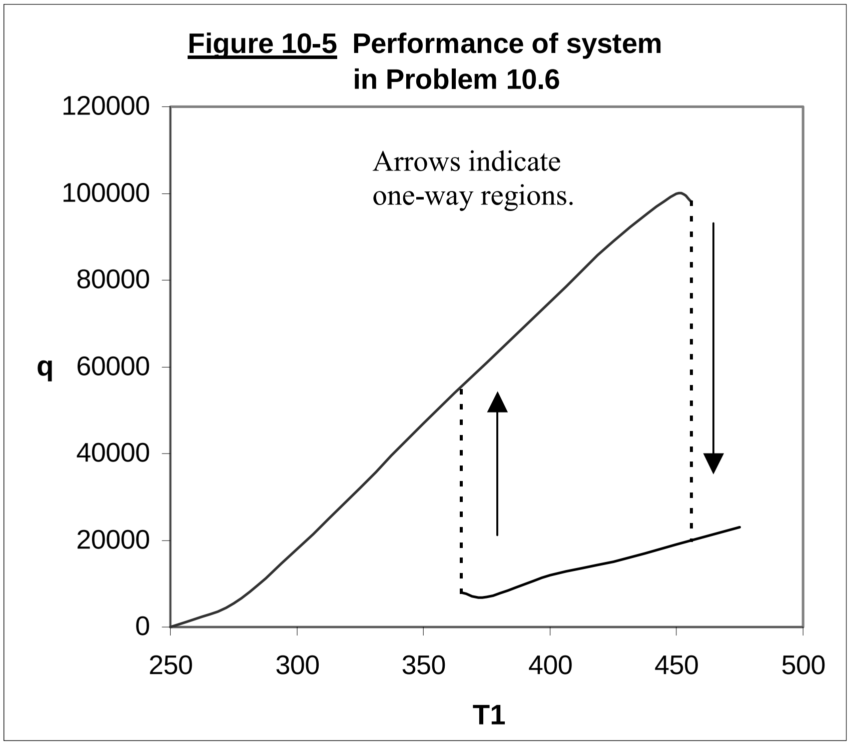
- Determine (q,T_1) coordinates at the intersections by inspection of Figure (10-4).

- Appraise the stability at each intersection by inspecting the intersection and applying Criterion (10-1).

- Plot the (q,T_1} coordinates of the stable intersections on Figure (10-5). Do not plot the coordinates of unstable intersections because the system will not remain at unstable intersections.

Problem 10.6 cont.

Figure (10-5) is the desired solution—a description of system performance $q\{T_1\}$ over the T_1 range of 250 to 475 F. Note the pronounced hysteresis when T_1 is in the range 365 to 455 F.



10.7 How to eliminate hysteresis

The system in Problem 10.6 can be modified to eliminate hysteresis in either of two ways:

- Adjust the T_1 controls so that T_1 cannot be increased above 440 F. This would prevent hysteresis, and would allow the system to operate at maximum heat flux.

- Modify the system so that the slope of the $q_{IN}\{T_{W2}\}$ lines in Figure (10-4) is more negative than the most negative slope of the $q_{OUT}\{T_{W2}\}$ curve.

Inspection of Figure (10-4) indicates that the most negative slope of the $q_{OUT}\{T_{W2}\}$ curve is approximately -1670 B/hrft^2. Therefore, in order to prevent hysteresis over the full range of the equipment, the modification must result in $dq_{IN}/dT_{W2} < -1670$. In other words, the constant in Eq. (10-9) must be increased to at least 1670.

The constant in Eq. (10-10) is determined by the heat transfer behavior of boundary layer 1 and the heat transfer wall. The former is described by Eq. (10-3), the latter by Eq. (10-4). Assuming that the heat transfer wall cannot be changed, the constant in Eq. (10-9) can be increased only by increasing the constant in Eq. (10-3). The required increase is determined in the following way:

- Let x be the value of the constant in Eq. (10-3) that will result in a constant of 1670 in Eq. (10-9).

- Substitute x for 830 in Eq. (10-7), and note from Figure (10-2) that $q_1 = q_W = q_{IN} = q$.

$$\Delta T_{IN} = q/x + q/11300 \qquad (10\text{-}11)$$

- Note that $dq_{IN}/dT_{W2} = -dq_{IN}/d\Delta T_{IN}$, and therefore

$$dq_{IN}/d\Delta T_{IN} = 1670 \qquad (10\text{-}12)$$

$$\therefore\ q_{IN} = 1670\ \Delta T_{IN} \qquad (10\text{-}13)$$

- Combine Eqs. (10-11) and (10-13).

$$q/1670 = q/x + q/11300 \qquad (10\text{-}14)$$

$$\therefore x = 1960$$

Therefore, hysteresis would be eliminated over the entire operating range of the system if boundary layer 1 were made to exhibit the behavior described by Eq. (10-15).

$$q_1 = 1960\ \Delta T_1 \qquad (10\text{-}15)$$

10.8 Problem 10.8—Undamped oscillation

Problem 10.8 differs from Problem 10.6 in that the system instability results in undamped oscillation as well as hysteresis. It should be noted that undamped oscillation results even though the heat source temperature and the heat sink temperature are constant.

Problem statement

Describe the performance of the heat transfer system in Figure (10-6) over the source fluid temperature range of 350 to 600 F. (Note that the system may be envisioned as a differential element in a vented pool boiler.)

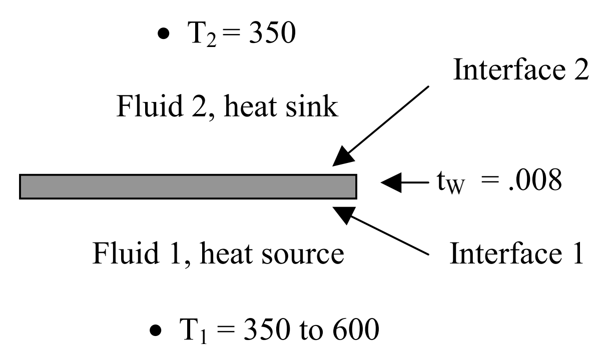


Figure 10-6 Heat transfer configuration, Problem 10.8

Given

- Equipment drawings.
- Identity of heat source fluid, heat sink fluid, and wall material.
- Flow rate of source fluid and sink fluid.
- The heat transfer behavior of boundary layer 1 is described by Eq. (10-16).

$$q_1 = .023\ (K/D)\ N_{Re}^{.8}\ N_{Pr}^{.4}\ \Delta T_1 \qquad (10\text{-}16)$$

Problem 10.8 cont.

- Evaluation of Eq. (10-16) gives Eq. (10-17).

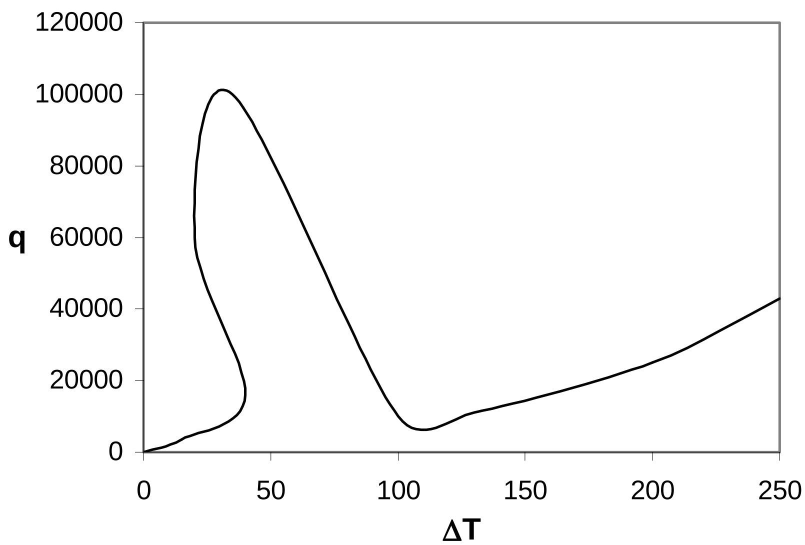
$$q_1 = 760\ \Delta T_1 \qquad (10\text{-}17)$$

- The heat transfer behavior of the wall material is described by Eq. (10-18).

$$q_W = 95\ \Delta T/t_W \qquad (10\text{-}18)$$

- The heat transfer behavior of boundary layer 2 is described by Figure (10-7).

Figure 10-7 Heat transfer behavior of Interface 2, Problem 10.8



Analysis—Determination of potential operating points

- Substitute in Eq. (10-18):

$$q_W = 95\ \Delta T_W/.008 = 11900\ \Delta T_W \qquad (10\text{-}19)$$

Problem 10.8 cont.

- Uncouple the system at the wall surface adjacent to Fluid 2, and determine $q_{IN}\{T_{W2}\}$ and $q_{OUT}\{T_{W2}\}$. (T_{W2} is the wall temperature adjacent to Fluid 2.)

- Determine $q_{IN}\{T_{W2}\}$ from inspection of Figure (10-6), and Eqs. (10-17) and (10-19).

$$T_1 - T_{W2} = \Delta T_1 + \Delta T_W \qquad (10\text{-}20)$$

$$T_1 - T_{W2} = q_1/760 + q_W/11900 \qquad (10\text{-}21)$$

$$q_1 = q_W = q_{IN} = q \qquad (10\text{-}22)$$

$$T_1 - T_{W2} = q/760 + q/11900 \qquad (10\text{-}23)$$

$$T_1 - T_{W2} = q_{IN}/714 \qquad (10\text{-}24)$$

$$q_{IN} = 714\,(T_1 - T_{W2}) \qquad (10\text{-}25)$$

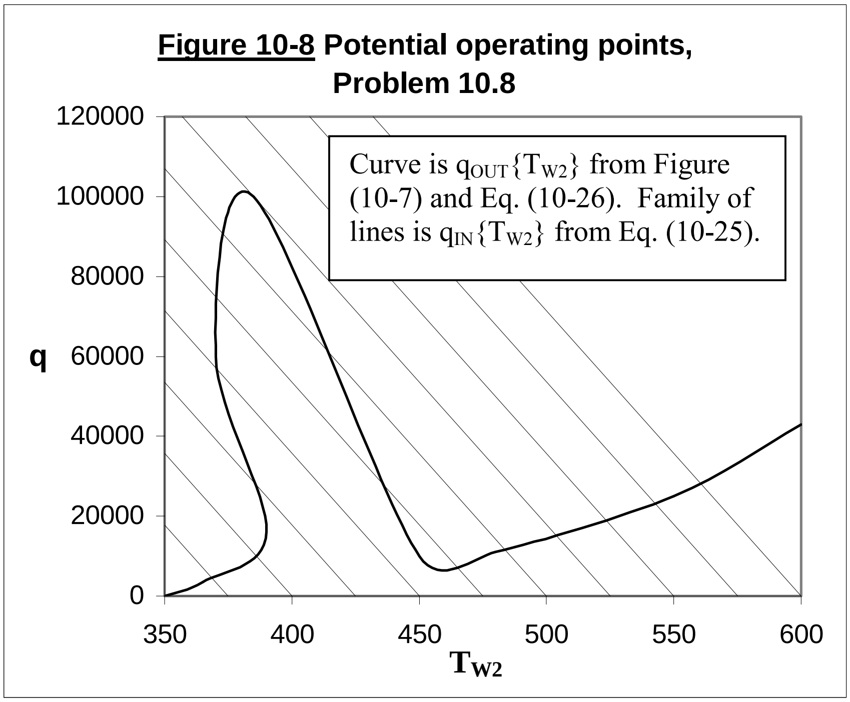
- Note that Figure (10-7) is in the form $q_{OUT}\{\Delta T_{OUT}\}$, and transform the figure to $q_{OUT}\{T_{W2}\}$ using Eq. (10-26). In other words, transform Figure (10-7) to the desired form by adding 350 to the coordinates on the x axis. The result is the curve in Figure (10-8).

$$T_{W2} = T_{SINK} + \Delta T_2 = 350 + \Delta T_2 \qquad (10\text{-}26)$$

- On Figure (10-8), plot Eq. (10-25) for various values of T_1 over its operating range of 350 to 600 F.

- Note that intersections in Figure (10-8) are potential operating points of the system.

Problem 10.8 cont.



Analysis—Stability at intersections

With regard to stability at the intersections in Figure (10-8), note the following:

- In Figure (10-8), $q_{OUT}\{T_{W2}\}$ includes a maximum and a minimum in $q_{OUT}\{T_{W2}\}$, *and* a maximum and a minimum in $T_{W2}\{q_{OUT}\}$.

- The maximum and minimum in $q_{OUT}\{T_{W2}\}$ occur at (q_{OUT},T_{W2}) coordinates of (101000,382) and (7000,460). The maximum and minimum in $T_{W2}\{q_{OUT}\}$ occur at (T_{W2},q_{OUT}) coordinates of (390,16000) and (370,67000).

- T_1 is constant along $q_{IN}\{T_{W2}\}$ lines in Figure (10-8). Eq. (10-25) indicates that $T_1 = T_{W2}$ at the x intercepts in Figure (10-8). In other words, $T_1 = T_{W2}\{q_{IN} = 0\}$. Therefore the value of T_1 on each of the $q_{IN}\{T_{W2}\}$ lines is readily determined by inspection of Figure (10-8).

Problem 10.8 cont.

- Note in Figure (10-8) that if T_1 = 410 to 460:
  - $q_{IN}\{T_{W2}\}$ intersects $q_{OUT}\{T_{W2}\}$ in the region between the maximum and minimum in $T_{W2}\{q_{OUT}\}$.
  - There is only one intersection on each $q_{IN}\{T_{W2}\}$ line, and it is unstable.
- Also note in Figure (10-8) that if T_1 = 465 to 525:
  - $q_{IN}\{T_{W2}\}$ intersects $q_{OUT}\{T_{W2}\}$ in the region between the maximum and minimum in $q_{OUT}\{T_{W2}\}$.
  - There are three intersections on each $q_{OUT}\{T_{W2}\}$ line, and only the middle intersections are unstable.

Analysis—behavior at unstable, singular solutions

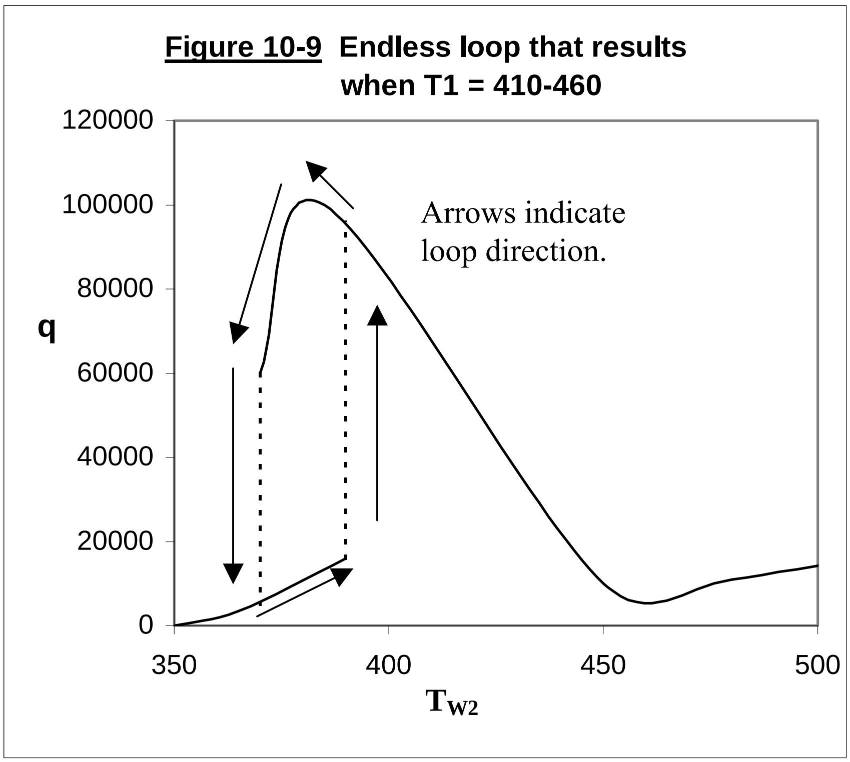
To determine the system behavior that results from an unstable, singular solution, refer to Figure (10-8), and suppose that the system is initially operating at the intersection on the q_{IN} line for T_1 = 425—ie the q_{IN} line that intercepts the x axis at T_{W2} = 425.

- The system suddenly receives a small, positive perturbation in T_{W2}.
- The positive perturbation causes q_{IN} to be larger than q_{OUT}. The mismatch between q_{IN} and q_{OUT} causes T_{W2} to increase.
- When T_{W2} increases to the maximum in $T_{W2}\{q_{OUT}\}$ at (390,16000), the mismatch between q_{IN} and q_{OUT} causes a step increase to (390,96000), since that is the only operating point at T_{W2} incrementally greater than 390.
- At (390,96000), q_{IN} is smaller than q_{OUT}. The mismatch between q_{IN} and q_{OUT} causes T_{W2} to decrease to the minimum in $T_{W2}\{q_{OUT}\}$ at (370,67000).
- At (370,67000), the mismatch between q_{IN} and q_{OUT} causes a step decrease to (370,5000), since that is the only operating point at T_{W2} incrementally smaller than 370.

Problem 10.8 cont.

- At (370,5000), q_{IN} is larger than q_{OUT}. The mismatch between q_{IN} and q_{OUT} causes T_{W2} to increase to the maximum in $T_{W2}\{q_{OUT}\}$ at (390,16000), and the cycle repeats.

As noted above, a single, unstable solution results when T_1 = 410 to 460. Therefore, when T_1 = 410 to 460, the system endlessly traverses the loop shown in Figure (10-9).

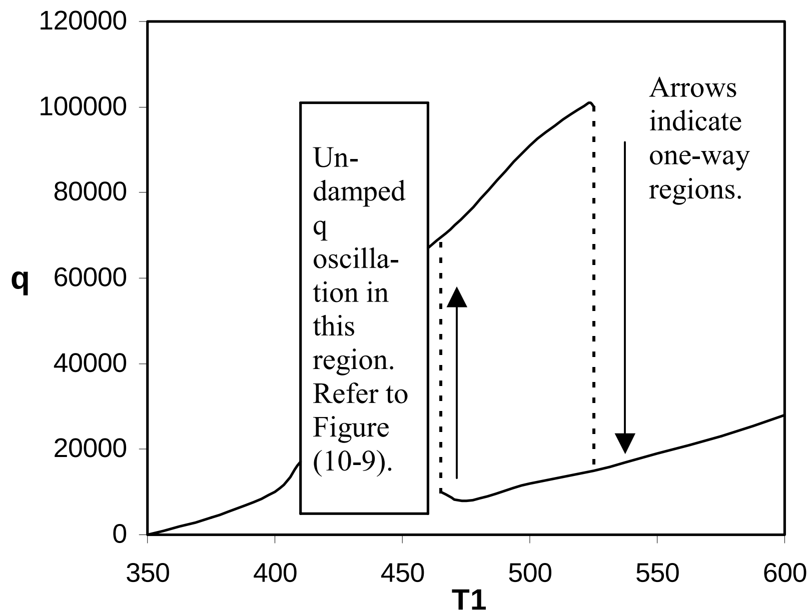


Solution of Problem 10.8

The solution of Problem 10.8 is a chart of $q\{T_1\}$. Coordinates of $q\{T_1\}$ are obtained from the intersections in Figure (10-8) by noting that $T_1 = T_{W2}\{q_{IN} = 0\}$. Unstable intersections are not plotted.

Figure (10-10) is the solution of Problem 10.8. It describes the system performance over the source fluid temperature range of 350 to 600.

Figure 10-10 Performance of system in Problem 10.8



10.9 How to eliminate undamped oscillation

Assuming that the behavior of boundary layer 2 cannot be modified, the undamped oscillation can be eliminated by modifying the system to make the $q_{IN}\{T_{W2}\}$ lines sufficiently steep to avoid singular, unstable solutions. In other words, the design objective is to make the lines sufficiently steep that all unstable intersections lie on $q_{IN}\{T_{W2}\}$ lines that make three intersections with $q_{OUT}\{T_{W2}\}$.

Note in Figure (10-8) that, if the $q_{IN}\{T_{W2}\}$ lines were steeper than any point on $T_{W2}\{q_{OUT}\}$ between the maximum and minimum in $T_{W2}\{ q_{OUT} \}$, all single intersections in this region would be replaced by three intersections, as desired.

In the region between the maximum and minimum in $T_{W2}\{ q_{OUT} \}$, the largest slope (ie the slope that is least steep) is –2000 B/hrft^{2}F. Therefore, if the system were modified so that the slope of the $q_{IN}\{T_{W2}\}$ lines were ≤ -2000 B/hrft^{2}F, the single intersections would be replaced by triple intersections, and the undamped oscillations would be replaced by hysteresis.

The slope of the $q_{IN}\{T_{W2}\}$ lines is determined by the behavior of boundary layer 1 and the heat transfer wall. To attain the desired slope, boundary layer 1 and/or the heat transfer wall must be modified so that the constant in Eq. (10-25) becomes equal to or greater than 2000.

If boundary layer 1 and/or the heat transfer wall were modified as required, the system performance would be affected as follows:

- Hysteresis would occur at heat source fluid temperatures in the vicinity of 390 F. The extent of the hysteresis depends on the behavior of the modified boundary layer 1 and heat transfer wall.
- The hysteresis that occurred in the original design (over the interval T_1 = 465 to 525) would be eliminated, since the steepest slope between the maximum and minimum in $q_{OUT}\{T_{W2}\}$ is –1520 B/hrft^{2}F.

10.10 Validation of the stability analysis in Problem 10.6

The stability analysis in Problem 10-6 is validated by results reported in Berenson’s pool boiling experiment (1960 and 1962), a benchmark experiment often cited in heat transfer literature.

The system in Problem 10.6 may be viewed as an incremental region in a pool boiler in which the heat source is a condensing fluid or a heated liquid, and the heat sink is a boiling liquid. This closely corresponds to the pool boiler in Berenson’s experiment.

Figure (10-5) is the result of the stability analysis of the pool boiler in Problem 10.6. The boiler contains a liquid that exhibits the “pool boiling curve” shown in Figure (10-3). The curve in Figure (10-3) resembles the generally accepted pool boiling curve for liquids such as those used in Berenson’s experiment. Therefore, if the stability analysis of Problem 10.6 is correct, the results of the stability analysis will agree with behavior reported by Berenson.

Note the following analytical results in Problem 10.6:

- The boiler may, or may not, be able to operate throughout the transition region of the pool boiling curve—ie throughout the region between the maximum and the minimum in Figure (10-3). The ability to operate throughout the transition region depends on the heat transfer behavior of the heat source boundary layer, the design of the heat transfer wall, and the boiling boundary layer.

- The boiler may, or may not, exhibit the pronounced hysteresis shown in Figure (10-5). The presence and extent of the hysteresis depends on the heat transfer behavior of the heat source boundary layer, the design of the heat transfer wall, and the boiling boundary layer.

Note that the following results reported by Berenson do in fact agree with the results of the stability analysis of Problem 10.6.

- Berenson reports the results of 20 runs that purport to "define the characteristic boiling curve completely". The runs included various boiling liquids and various treatments of the boiling surface. Of these runs, 3 contain data throughout the transition region, and 17 contain essentially no data in the transition region. The lack of data in 17 runs surely reflects an inability to obtain the data, since the primary purpose of the experiment was to investigate transition boiling, as reflected in the title of Berenson's thesis.

- Berenson does not report the temperature of the heat source fluid. However, if the temperature of the heat source fluid is estimated from information given in the thesis, it is evident that the boiler exhibited pronounced hysteresis in the 17 runs that contained essentially no data in the transition region

(For a more comprehensive treatment of this subject, see Adiutori, 1991.)

10.10.2 Validation of the stability analysis in Problem 10.8

The stability analysis in Problem 10.8 is validated by results reported in Marto and Rohsenow (1966). Their results were obtained from a pool boiler in which the boiling fluid was a liquid metal.

The pool boiler in Problem 10.8 contains a liquid that exhibits the "pool boiling curve" shown in Figure (10-7). Adiutori (1964) suggests that the pool boiling curve for liquid metals resembles the curve in Figure (10-7).

(It is not yet widely accepted that the pool boiling curve for liquid metals resembles Figure (10-7).)

If the stability analysis of Problem 10.8 is correct, and if in fact the pool boiling curve for liquid metals resembles Figure (10-7), then behavior predicted in the analysis will agree with behavior reported by Marto and Rohsenow.

Note the following analytical results in Problem 10.8:

- As indicated in Figures (10-9) and (10-10), the boiler exhibits undamped, oscillatory behavior when operated near the lower end of its operating range.
- As indicated in Figure (10-9), the undamped oscillation includes periods in which boiling occurs, and periods in which boiling does not occur. (In Figure (10-9), there is no boiling on the bottom leg of the loop. In this region, heat transfer is by convection in the liquid, and evaporation at the free interface. Inexplicably, boiling does not occur at all points of the widely accepted "pool boiling curve".)
- As indicated in Figure (10-9), when boiling occurs, the wall temperature decreases with time. When boiling does not occur, the wall temperature increases with time.
- As indicated in Figure (10-10), when boiler operation is brought into the upper region of its operating range, the undamped oscillations cease, and the boiler operates in a steady manner.

Note that the following observations by Marto and Rohsenow do in fact agree with the stability analysis of Problem 10.8.

> During nucleation, large boiler wall temperature fluctuations occurred which in some cases were as high as 150 F . . .
>
> These fluctuations were always accompanied by large variations in the test section noise level as determined from the phonograph cartridge. The sharp increase in noise level and the sudden decrease in wall temperature of the boiler always occurred coincidentally . . . This is interpreted to be the onset of nucleate boiling. After this "bump", nucleation may continue . . . as evidenced by the continued noise level and lower wall superheat . . . When the noise stops, the temperature rises gradually to its maximum value.

> When boiling is stable, the wall temperature remains at the lower level and the noise persists.
>
> All the unstable data show that, as the heat flux is increased, stability improves . . . The experiment results show that, around 200,000 $B/hrft^2$, stable boiling occurs in most cases.

10.11 Summary of heat transfer science in the new engineering

Heat transfer science in the new engineering is summarized by the following:

- q, T, ΔT, and their derivatives remain *separate* and *explicit*.
- The words convective and conductive are retained. They have the same meaning they have in conventional engineering.
- Convective heat transfer phenomena are described, analyzed, and predicted using convective thermal behavior q{ΔT}. For example, $q = 3\ \Delta T^{1.33}$ is used in place of $h = 3\ \Delta T^{0.33}$ $Btu/hrft^2F$.
- Conductive heat transfer phenomena are described, analyzed, and predicted using conductive thermal behavior q{dT/dx}. For example, q = 11 (dT/dx) is used in place of *k = 11 Btu/hrftF*.
- Eqs. (10-27) and (10-28) are abandoned. They are replaced by Eqs. (10-29) and (10-30).

$$q_{conv} = h\ \Delta T \qquad (10\text{-}27)$$

$$q_{cond} = k\{T\}(\ dT/dx) \qquad (10\text{-}28)$$

$$q_{conv} = f\{\Delta T\} \qquad (10\text{-}27)$$

$$q_{cond} = f\{T,\ dT/dx\} \qquad (10\text{-}28)$$

- The parameters heat transfer coefficient and thermal conductivity are abandoned.
- The words coefficient and conductivity are abandoned.
- The symbols *h* and *k* are abandoned.

Chapter 11

Example problems that illustrate stress/strain analysis using behavior methodology

11 Introduction

This chapter includes example problems that illustrate stress/strain analysis using stress/strain "behavior" methodology—ie methodology in which stress (σ) and strain (ε) are *separate* and *explicit*. The problems include proportional and nonlinear phenomena, and demonstrate that stress/strain analysis is simple and direct using behavior methodology.

The problems in this chapter and in Chapter 12 are based on an idealized material. It is idealized in that operation over the entire stress/strain curve is reversible, and therefore the material is not subject to permanent strain. The material is idealized so that the analyses in this chapter will be closely analogous to electrical and heat transfer analyses in previous chapters. The impact of permanent strain is addressed in Chapter 14.

In Chapter 12, the problems in this chapter are restated (but not solved) using modulus methodology. The reader is encouraged to solve the problems using modulus methodology, and to compare her/his modulus analyses with the behavior analyses presented in this chapter.

The reader will find that the proportional problems are easy to solve with behavior methodology or modulus methodology because both allow direct solution. The reader will also find that the nonlinear problems are *much* easier to solve using behavior methodology because it allows direct solution of nonlinear problems, whereas modulus methodology generally requires indirect solution of nonlinear problems.

11.1 The relationship between σ and ε

The relationship between σ and ε is determined empirically, and the data are used to prepare charts in the form σ{ε}. Figure (11-1) describes a more or less typical relationship between σ and ε.

Note in Figure (11-1) that, at small values of strain, the stress is proportional to the strain. This region is referred to as the elastic region.

Also note in Figure (11-1) that, at large values of strain, stress is a highly nonlinear function of strain. This region is referred to as the inelastic region

11.2 Stress/strain "modulus", the ratio σ/ε

In conventional engineering, the analysis of stress/strain problems is based on the ratio σ/ε. This ratio is assigned the name "modulus" and the symbol E.

- Modulus values are obtained by dividing σ data by ε data. Figure (11-1M) is the result of transforming the data in Figure (11-1). Note that the two figures are *identica*l. They differ only in form. Figure (11-1) is in the raw data form σ{ε}, whereas Figure (11-1M) is in the modulus form $(\sigma/\varepsilon)\{\varepsilon\}$—ie the form $E\{\varepsilon\}$.

- When σ/ε (symbol E) is used to describe elastic behavior, it is called elastic modulus. It is empirically determined by measuring the value of σ/ε at a small value of ε (usually .002). In the elastic region, equations and charts indicate that σ/ε is independent of ε at small values of ε—*ie* that E is independent of ε at small values of ε. Equation (11-1) and Figure (11-1M) are typical.

$$\sigma/\varepsilon = E = 30 \times 10^6 \text{ psi} \qquad \text{for } \varepsilon < .002 \tag{11-1}$$

- When σ/ε (symbol E) is used to describe inelastic behavior, it is called "plastic" modulus or "secant" modulus. In the inelastic region, equations and charts indicate that σ/ε (symbol E) is a non-linear function of ε at large values of ε. Figure (11-1M) is typical.

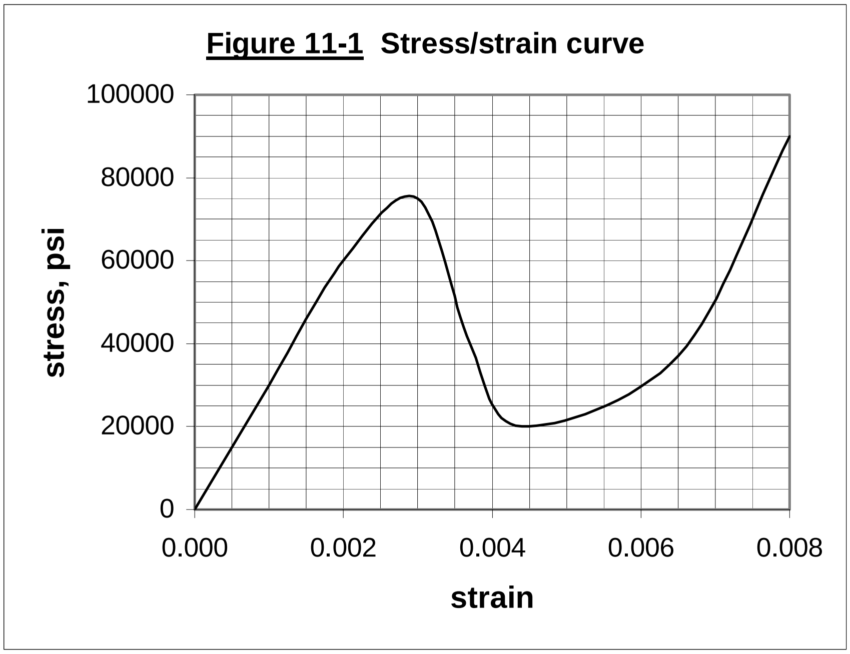


Figure 11-1 Stress/strain curve
stress, psi
100000
80000
60000
40000
20000
0
0.000
0.002
0.004
0.006
0.008
strain

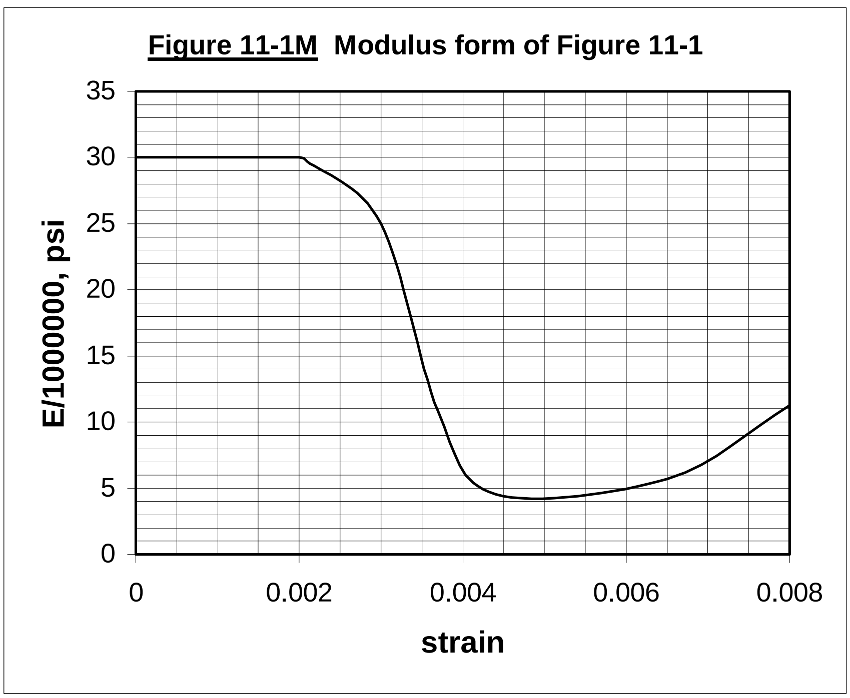


Figure 11-1M Modulus form of Figure 11-1
E/1000000, psi
35
30
25
20
15
10
5
0
0
0.002
0.004
0.006
0.008
strain

11.3 Stress/strain "behavior", the function $\sigma\{\varepsilon\}$

In behavior methodology, the analysis of stress/strain problems is based on the "behavior" of σ and ε—ie is based on the function $\sigma\{\varepsilon\}$. Note that

- Stress/strain data are obtained in the raw data form $\sigma\{\varepsilon\}$.
- The raw data form $\sigma\{\varepsilon\}$ is also the behavior form—ie is also the form required for analysis in behavior methodology.
- Eq. (11-2) and Figure (11-1) are examples of the behavior form $\sigma\{\varepsilon\}$

$$\sigma = 30 \times 10^6\ \varepsilon \qquad \text{for } \varepsilon < .002 \tag{11-2}$$

11.4 Modulus methodology and behavior methodology

- In modulus methodology, the relationship between σ and ε is described in the form $\sigma/\varepsilon\{\varepsilon\}$—ie the form $E\{\varepsilon\}$, as in Eq. (11-1) and Figure (11-1M). Because σ and ε are combined in E (the symbol for σ/ε), analyses based on modulus methodology must be performed with σ and ε *combined*.
- In behavior methodology, the relationship between σ and ε is described in the form $\sigma\{\varepsilon\}$, as in Figure (11-1) and Eq. (11-2). Because σ and ε are separated, analyses based on behavior methodology can be performed with σ and ε *separated*.

Modulus methodology and behavior methodology are *identical*—they differ only in form. In modulus methodology, σ and ε are combined. In behavior methodology, σ and ε are separated. For example, Eqs. (11-1) and (11-2) are identical, and differ only in form. Figures (11-1) and (11-1M) are also identical, and differ only in form.

The particular advantage of behavior methodology is that it greatly simplifies the solution of nonlinear stress/strain problems in general because it allows problems to be solved with the variables separate. Modulus methodology requires that problems be solved with the variables combined, resulting in indirect and much more difficult solutions.

11.5 Problem 11.5—Proportional phenomena

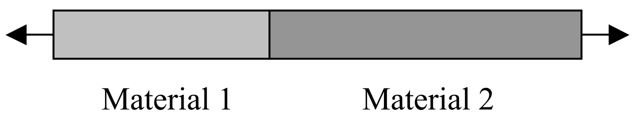
Problem 11.5 serves two purposes:

- It demonstrates how to use behavior methodology to solve stress/strain problems that concern proportional phenomena—ie it demonstrates how to solve proportional problems with σ and ε separated rather than combined in the ratio σ/ε (symbol E).

- It demonstrates that the solution of proportional problems based on behavior methodology is as simple as solution based on modulus methodology.

Problem statement

What axial load would increase the length of the bar below by .005 feet? What stress and strain would result in each material?



Given

- Material 1 is 2 feet long, and its behavior is described by Eq. (11-3).

$$\sigma_1 = 25 \text{ x } 10^6 \varepsilon_1 \qquad \text{for } \varepsilon_1 < .002 \qquad (11\text{-}3)$$

- Material 2 is 3 feet long, and its behavior is described by Eq. (11-4).

$$\sigma_2 = 40 \text{ x } 10^6 \varepsilon_2 \qquad \text{for } \varepsilon_2 < .002 \qquad (11\text{-}4)$$

- The cross-section of the bar is everywhere 4 in^2.

Analysis

- From inspection of the figure and the given information,

$$\sigma_1 = \sigma_2 = \sigma \qquad (11\text{-}5)$$

$$2\varepsilon_1 + 3\varepsilon_2 = .005 \qquad (11\text{-}6)$$

Problem 11.5 cont.

- Substituting the given information in Eq. (11-6), and using Eq. (11-5), gives Eq. (11-7).

$$2\sigma/(25\text{x}10^{6}) + 3\sigma/(40\text{x}10^{6}) = .005 \qquad (11\text{-}7)$$

- Solution of Eq. (11-7) gives σ = 32,300.

- Since the cross-sectional area of the bar is 4 in^2, the 32,300 psi stress indicates that a load of 129,200 lbs would increase the length of the bar by .005 ft.

- Eq. (11-3) indicates that a stress of 32,300 psi causes a strain of 0.0013 in Material 1.

- Eq. (11-4) indicates that a stress of 32,300 psi causes a strain of 0.00081 in Material 2.

Solution

- A load of 129,200 lbs would increase the length of the bar by .005 ft.

- The load would result in a stress of 32,300 psi in both materials.

- The strain in Material 1 would be .0013. The strain in Material 2 would be .00081.

Points to consider in Problem 11.5

- The problem is stated and solved using behavior methodology—ie with σ and ε separate and explicit.

- The problem is stated and solved *without E*, demonstrating that *E* is unnecessary.

- The solution based on behavior methodology is simple and direct.

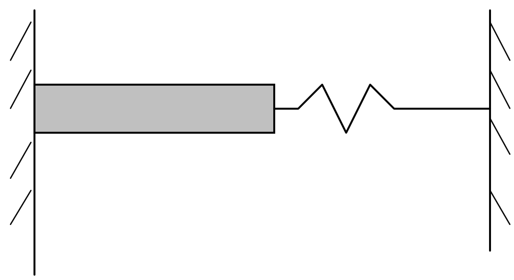
11.6 Problem 11.6—Proportional phenomena

This problem demonstrates the application of behavior methodology to a stress/strain problem that concerns a bar and a spring. Because both components exhibit proportional behavior, the solution of the problem is quite simple.

Problem 11.8 is the same as this problem except that it concerns nonlinear behavior. Together the problems demonstrate that, using behavior methodology, nonlinear problems and proportional problems are analyzed in the same simple and direct manner.

Problem statement

What are the stress and strain values in the bar below?



Given:

- No load length of bar = 4.950 ft.

- Bar cross-section = 4 in^2

- No load length of spring = 2.000 ft.

- Distance between walls = 7.000 ft

- The behavior of the spring is described by Eq. (11-8). (P is load in pounds, L is length in feet.)

$$P_{spring} = 6 \text{ x } 10^6 \, \Delta L_{spring} \tag{11-8}$$

Problem 11.6 cont.

- The behavior of the bar material is described by Eq. (11-9).

$$\sigma_{bar} = 30 \text{ x } 10^{6}\varepsilon_{bar} \qquad \text{for } \varepsilon_{bar} < .0025 \qquad (11\text{-}9)$$

Analysis

The analysis involves the following:

- Note that $P_{bar} = P_{spring}$.
- Obtain expressions for $(P_{bar})\{\varepsilon_{bar}\}$ and $(P_{spring})\{\varepsilon_{bar}\}$.
- Equate $(P_{bar})\{\varepsilon_{bar}\}$ and $(P_{spring})\{\varepsilon_{bar}\}$, and solve for $\varepsilon_{bar.}$

The expression for $(P_{bar})\{\varepsilon_{bar}\}$ is obtained by noting that

$$(P_{bar}) = \sigma_{bar}\, A_{bar} \qquad (11\text{-}10)$$

Substituting given information in Eq. (11-10) gives

$$(P_{bar})\{\varepsilon_{bar}\} = 120 \text{ x } 10^{6}\varepsilon_{bar} \qquad \text{for } \varepsilon_{bar} < .0025 \qquad (11\text{-}11)$$

The expression for $(P_{spring})\{\varepsilon_{bar}\}$ is obtained by calculating the coordinates of two points on $(P_{spring})\{\varepsilon_{bar}\}$, and noting that the spring load is a linear function of ε_{bar}.

The coordinates of one point are calculated by noting that

$$\Delta L_{spring}\{\varepsilon_{bar} = 0\} = .05 \qquad (11\text{-}12)$$

$$\therefore\ P_{spring}\{\varepsilon_{bar} = 0\} = .05 \text{ x } 6 \text{ x } 10^{6} = 300{,}000 \qquad (11\text{-}13)$$

The coordinates of a second point are calculated by noting that

$$\Delta L_{spring}\{\varepsilon_{bar} = .01\} = 0 \qquad (11\text{-}14)$$

$$\therefore\ P_{spring}\{\varepsilon_{bar} = .01\} = 0 \qquad (11\text{-}15)$$

Problem 11.6 cont.

Since $(P_{spring})\{\varepsilon_{bar}\}$ is a linear function, Eqs. (11-11) and (11-15) indicate that the function is described by Eq. (11-16).

$$(P_{spring})\{\varepsilon_{bar}\} = 300{,}000 - 30 \times 10^6\, \varepsilon_{bar} \qquad (11\text{-}16)$$

Equating $(P_{bar})\{\varepsilon_{bar}\}$ and $(P_{spring})\{\varepsilon_{bar}\}$ from Eqs. (11-11) and (11-16) gives

$$120 \times 10^6 \varepsilon_{bar} = 300{,}000 - 30 \times 10^6\, \varepsilon_{bar} \qquad (11\text{-}17)$$

Solution of Equation (11-17) indicates that $\varepsilon_{bar} = .0020$. Therefore Eq. (11-9) applies, and indicates that $\sigma_{bar} = 60{,}000$.

Solution

The stress in the bar is 60,000 psi. The strain in the bar is .0020.

Points to consider in Problem 11.6

- The problem is stated and solved using behavior methodology—ie with σ and ε separate and explicit.
- The problem is stated and solved *without E*, demonstrating that *E* is unnecessary.
- The solution based on behavior methodology is simple and direct.

11.7 Problem 11.7—Nonlinear component

This problem demonstrates the application of behavior methodology to a stress/strain problem that concerns the highly nonlinear behavior of a single component. Because the problem involves only a single component, the solution is so simple as to seem trivial.

However, the solution of Problem 11.7 using modulus methodology is far from trivial. The reader will find that it takes approximately 100 times longer to solve Problem 11.7 using modulus methodology, and the likelihood of error is at least 100 times greater.

Problem statement

Given the behavior described in Figure (11-1), what strain would result from a stress of 40,000 psi?

Analysis and solution

Inspection of Figure (11-1) indicates that a stress of 40,000 psi would result in a strain of .0013, .0037, or .0066 in the subject material. The problem statement does not contain sufficient information to obtain a unique answer.

Points to consider in Problem 11.7

- Even though Problem 11.7 concerns highly nonlinear behavior, the solution based on behavior methodology is so simple that the problem seems trivial.

- Using modulus methodology, the problem is far from trivial. Note that the modulus statement of Problem 11.7 is:

  Given the behavior described in Figure (11-1M), what strain would result from a stress of 40,000 psi?

  Note that the solution cannot be obtained by simple inspection of Figure (11-1M). The figure must be read in an indirect manner because stress appears only in implicit form.

- Even though the problem seems trivial using behavior methodology, it accurately reflects the difference between behavior methodology and modulus methodology.

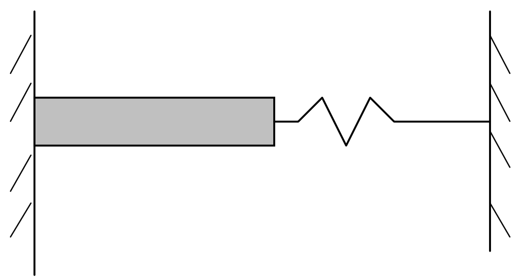
11.8 Problem 11.8—System with a nonlinear component

This problem demonstrates the application of behavior methodology to a problem that concerns a highly nonlinear component in a system. The problem is identical to Problem 11.6 except that the behavior of the bar is highly nonlinear. In spite of the highly nonlinear behavior and the several components, the solution of the problem based on behavior methodology is simple and direct.

In particular, note that this highly nonlinear problem is analyzed in the same simple and direct manner as its proportional counterpart, Problem 11.6. The sole difference is that the analysis of Problem 11.6 is digital, whereas the analysis of this problem is necessarily graphical because the behavior of the nonlinear component is described graphically.

Problem statement

What are the steady-state stress and strain values in the bar below?



Given

- No load length of bar = 4.950 ft.
- Bar cross-section = 4 in^2
- No load length of spring = 2.000 ft.
- Distance between walls = 7.000 ft
- The behavior of the spring is described by Eq. (11-8).
- The behavior of the bar material is described by Figure (11-1).

Problem 11.8 cont.

Analysis

The analysis is the same as the analysis of Problem 11.6 except that the focus is on stress rather than load because Figure (11-1) describes the stress behavior of the bar.

- Note that $P_{bar} = P_{spring}$.
- Obtain expressions for $(P_{bar}/A_{bar})\{\varepsilon_{bar}\}$ and $(P_{spring}/A_{bar})\{\varepsilon_{bar}\}$.
- Equate $(P_{bar}/A_{bar})\{\varepsilon_{bar}\}$ and $(P_{spring}/A_{bar})\{\varepsilon_{bar}\}$, and solve for $\varepsilon_{bar.}$

$(P_{bar}/A_{bar})\{\varepsilon_{bar}\}$ is described graphically in Figure (11-1). The expression for $(P_{spring}/A_{bar})\{\varepsilon_{bar}\}$ is obtained from Eq. (11-17) by dividing all terms by A_{bar}.

$$30 \times 10^{6}\varepsilon_{bar} = 75{,}000 - 7.5 \times 10^{6}\,\varepsilon_{bar} \qquad (11\text{-}18)$$

Graphically equate $(P_{bar}/A_{bar})\{\varepsilon_{bar}\}$ and $(P_{spring}/A_{bar})\{\varepsilon_{bar}\}$ by plotting them together in Figure (11-2), and noting that intersections are solutions.

Solution

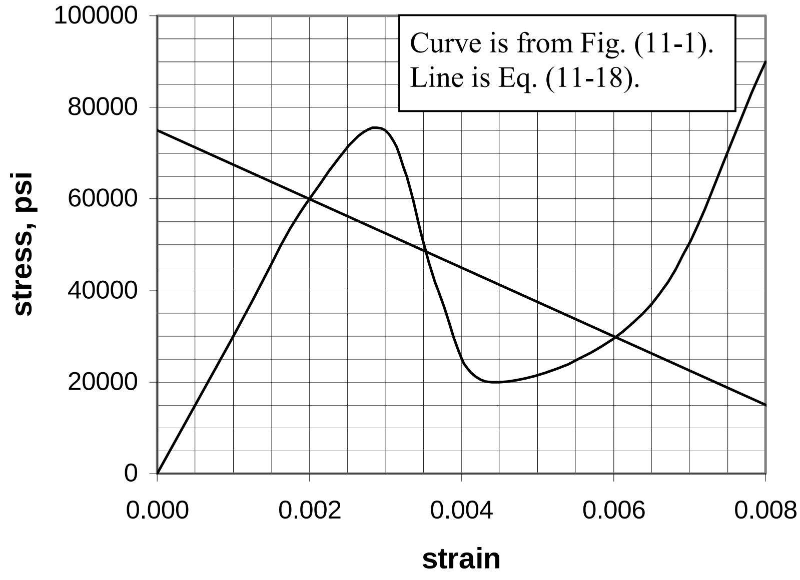
Figure (11-2) indicates that there are three possible solutions to the problem:

Stress, psi	**Strain**
60,000	.0020
49,000	.0035
30,000	.0060

Inspection of Figure (11-2) indicates that the solution at 49,000 psi is unstable. (See Section 14.10.) Because of the instability, the system would not operate at 49,000 psi, but would simply shift to the solution at 30,000 psi.

Therefore the steady-state stress and strain values are 60,000 psi at .0020 strain, and 30,000 psi at .0060 strain. The problem statement does not contain sufficient information to determine a unique solution.

Figure 11-2 Solution of Problem 11.8



Points to consider in Problem 11.8

- The problem is stated and solved using behavior methodology—ie with σ and ε separate and explicit.

- The problem is stated and solved *without E*, demonstrating that *E* is unnecessary.

- The solution based on behavior methodology is simple and direct, even though the problem concerns highly nonlinear behavior, and several components.

- The behavior analysis of Problem 11.8 (an inelastic problem) is exactly the same as the behavior analysis of Problem 11.6 (an elastic problem). This illustrates that, if behavior methodology is used, elastic problems and inelastic problems are analyzed in the same direct way. On the other hand, if modulus methodology is used, elastic problems can be solved in a direct manner, but inelastic problems must generally be solved in an indirect manner.

11.9 Closing remarks

The problems in this chapter are intended to:

- Illustrate how to use behavior methodology to solve stress/strain problems that involve elastic and/or inelastic behavior. In other words, illustrate how to solve problems with σ and ε *separate* and *explicit*, and *without E*.

- Demonstrate that problems that involve elastic behavior and/or inelastic behavior are solved simply and directly using behavior methodology.

- Demonstrate that it is not necessary to solve elastic or inelastic stress/strain problems using the ratio σ/ε assigned the name "modulus" and the symbol *E*.

By themselves, the problems in this chapter do not demonstrate that nonlinear stress/strain problems are easier to solve using behavior methodology rather than modulus methodology. That demonstration must be performed by the reader—by solving the problems in the next chapter using modulus methodology.

The problems in the next chapter are identical to the problems in this chapter—they differ only in form. The problems in this chapter are presented in behavior form; the problems in the next chapter are presented in modulus form.

Only by comparing his/her analyses of the nonlinear problems in the next chapter with the analyses presented in this chapter will the reader gain a first hand knowledge of the simplicity that results from behavior methodology relative to modulus methodology.

Chapter 12

The modulus form of the problems in Chapter 11

12 Introduction

In Chapter 11, stress/strain problems are stated and solved using *behavior* methodology. In this chapter, the problems in Chapter 11 are restated (but not solved) using *modulus* methodology

The reader is encouraged to solve the problems in this chapter using modulus methodology. By comparing his/her modulus analyses with the behavior analyses in Chapter 11, the reader will find that the proportional problems are solved in a simple and direct manner using behavior methodology or modulus methodology.

The reader will also find that the nonlinear problems are much easier to solve using behavior methodology because it allows direct solution of nonlinear problems, whereas modulus methodology generally requires indirect solution of nonlinear problems.

The problems, figures, and equations in this chapter are *identical* to those in Chapter 11. They differ only in form. The behavior form is used throughout Chapter 11, the modulus form is used throughout this chapter.

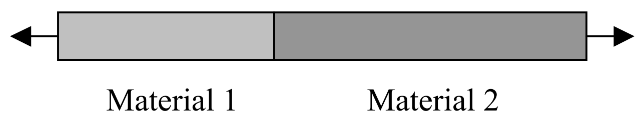
Corresponding problems, figures, and equations in this chapter have the same identifying numbers used in Chapter 11, except that "M" is added to the identifying numbers in this chapter (to denote modulus form). For example, Problem 11.5M is the modulus form of Problem 11.5 in Chapter 11. Eq. (11-3M) is the modulus form of Eq. (11-3) in Chapter 11.

12.1 Problem 11.5M Proportional phenomena

This problem is the modulus form of Problem 11.5. The problems are identical except for form.

Problem statement

What axial load would increase the length of the bar below by .005 feet? What stress and strain would result in each material?



Given

Material 1 is 2 feet long. Its σ{ε} behavior is described by Eq. (11-3M).

$$E_1 = 25 \text{ x } 10^6 \text{ psi} \qquad \text{for } \varepsilon_1 < .002 \qquad (11\text{-}3M)$$

Material 2 is 3 feet long Its σ{ε} behavior is described by Eq. (11-4M).

$$E_2 = 40 \text{ x } 10^6 \text{ psi} \qquad \text{for } \varepsilon_2 < .002 \qquad (11\text{-}4M)$$

The cross-section of the bar is everywhere 4 in^2.

Analysis and solution

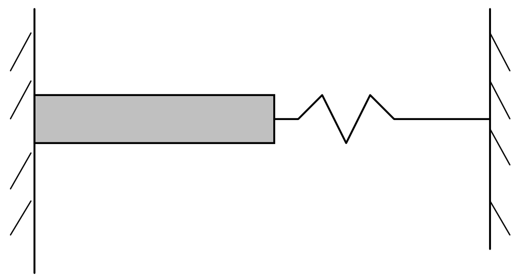
(To be determined by the reader.)

12.2 Problem 11.6M Proportional phenomena

This problem is the modulus form of Problem 11.6. The problems are identical except for form.

Problem statement

What are the stress and strain values in the bar below?



Given

- No load length of bar = 4.950 ft.
- Bar cross-section = 4 in^2
- No load length of spring = 2.000 ft.
- Distance between walls = 7.000 ft
- Spring constant = 6 x 10^6 lbs/ft
- The modulus of the bar material is described by Eq. (11-9M).

$$E_{bar} = 30 \times 10^6 \qquad \text{for } \varepsilon_{bar} < .0025 \qquad \text{(11-9M)}$$

Analysis and solution

To be determined by the reader.

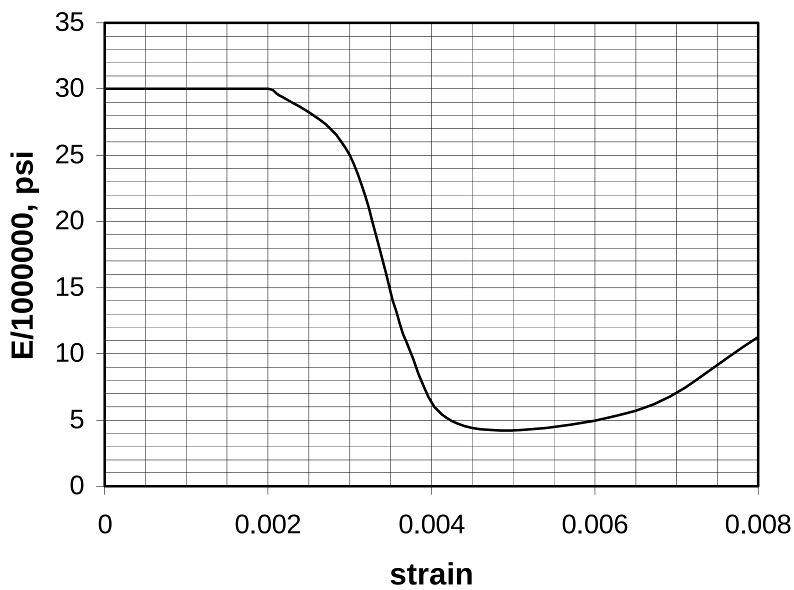
12.3 Problem 11.7M Nonlinear component

This problem is the modulus form of Problem 11.7. The problems are identical except for form.

Problem statement

Given the modulus described in Figure (11-1M), what strain would result from a stress of 40,000 psi?

Figure 11-1M Modulus form of Figure 11-1



Analysis and solution

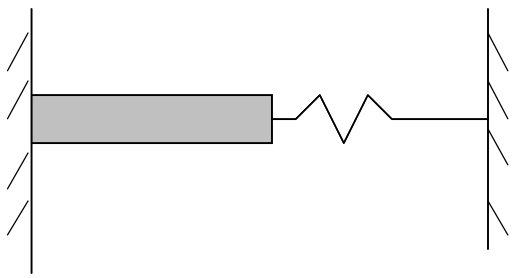
To be determined by the reader.

12.4 Problem 11.8M System with a nonlinear component

This problem is the modulus form of Problem 11.8. The problems are identical except for form.

Problem statement

What are the stress and strain values in the bar below?



Given

- No load length of bar = 4.950 ft.
- Bar cross-section = 4 in^2
- No load length of spring = 2.000 ft.
- Distance between walls = 7.000 ft
- Spring constant = 6 x 10^6 lbs/ft
- The modulus of the bar material is described by Figure (11-1M).

Analysis and solution

To be determined by the reader.

12.5 Closing remarks

The problems in Chapters 11 and 12 illustrate that nonlinear stress/strain problems are easier to solve using behavior methodology rather than modulus methodology.

Modulus methodology complicates the solution of nonlinear problems because the variables σ and ε are combined in the ratio σ/ε assigned the name "modulus" and the symbol E. This generally makes it necessary to solve nonlinear problems in an indirect and unnecessarily difficult manner, whereas they can be solved in a direct and much simpler manner if σ and ε are separate.

The variables σ and ε can be separated by abandoning modulus and modulus methodology, and replacing them with behavior and behavior methodology.

The reader is encouraged to attempt to solve all the problems in this chapter using modulus methodology in order to become convinced that

- Behavior methodology is preferable to modulus methodology.

- Modulus and modulus methodology can and should be abandoned.

- Modulus and modulus methodology should be replaced by behavior and behavior methodology.

Chapter 13

Why stress/strain behavior σ{ε} should replace stress/strain modulus σ/ε

13 Introduction

This chapter addresses the question

Should stress strain "behavior" replace stress/strain "modulus"?

The question is answered in two ways:

- In a general way by critically examining the nature and application of "modulus".
- In a specific way by comparing the behavior analyses in Chapter 11 with the modulus analyses of the same problems in Chapter 12.

The answers strongly support the conclusion that stress/strain "behavior" should replace stress/strain "modulus".

13.1 The ratio named "modulus"

In conventional engineering, the ratio σ/ε is named "modulus", and is usually assigned the symbol E. This ratio is used in stress/strain analyses to describe the relationship between σ and ε.

Every modulus value is the result of a test to measure the relationship between stress and strain—*ie* to measure the function $\sigma\{\varepsilon\}$. Stress/strain data are reduced to several different moduluses. For example,

- Elastic modulus is usually taken to be the measured value of stress divided by the measured value of strain at a strain of .002. Symbolically, $E_{elastic}$ is the measured value of σ/ε at $\varepsilon = .002$.
- Plastic or secant modulus is the measured value of stress divided by the measured value of strain at any value of strain. Symbolically, $E_{plastic}$ is the measured value of σ/ε at any value of ε.

13.2 The rationale of reducing stress/strain data to modulus

In modulus methodology:

- Tests are performed to determine the relationship between stress and strain—ie to determine $\sigma\{\varepsilon\}$.
- Modulus values are determined by transforming $\sigma\{\varepsilon\}$ data to the form $\sigma/\varepsilon\{\varepsilon\}$—*ie* to the form $E\{\varepsilon\}$.
- $\sigma/\varepsilon\{\varepsilon\}$ is used in stress analyses to describe $\sigma\{\varepsilon\}$—*ie* $E\{\varepsilon\}$ is used in stress analyses to describe $\sigma\{\varepsilon\}$.

At this point, it is appropriate to ask:

- Since stress/strain data are obtained in the form $\sigma\{\varepsilon\}$, is it really desirable to transform $\sigma\{\varepsilon\}$ data to the form $(\sigma/\varepsilon)\{\varepsilon\}$ since the only purpose of $(\sigma/\varepsilon)\{\varepsilon\}$ is to describe $\sigma\{\varepsilon\}$?
- Isn't $(\sigma/\varepsilon)\{\varepsilon\}$ a poor way to describe $\sigma\{\varepsilon\}$?
- Isn't $\sigma\{\varepsilon\}$ the best way to describe $\sigma\{\varepsilon\}$?
- Since $\sigma\{\varepsilon\}$ is the best way to describe $\sigma\{\varepsilon\}$, wouldn't it be better to use $\sigma\{\varepsilon\}$ in stress/strain analyses, and to abandon $(\sigma/\varepsilon)\{\varepsilon\}$?

The appropriate responses are:

- Since stress/strain data are obtained in the form $\sigma\{\varepsilon\}$, it is a pointless exercise to transform $\sigma\{\varepsilon\}$ data to the form $(\sigma/\varepsilon)\{\varepsilon\}$, since the only purpose of $(\sigma/\varepsilon)\{\varepsilon\}$ is to describe $\sigma\{\varepsilon\}$.
- $(\sigma/\varepsilon)\{\varepsilon\}$ is a very poor way to describe $\sigma\{\varepsilon\}$. Using $(\sigma/\varepsilon)\{\varepsilon\}$ to describe $\sigma\{\varepsilon\}$ is like using $(y/x)\{x\}$ to describe $y\{x\}$.
- Surely it is a truism to say that $\sigma\{\varepsilon\}$ is the best way to describe $\sigma\{\varepsilon\}$.
- Yes!

13.3 The nature of stress/strain "behavior"

Stress/strain "behavior" is $\sigma\{\varepsilon\}$. It is determined empirically by performing stress/strain tests over the range zero load to fracture, and then plotting the data in the form $\sigma\{\varepsilon\}$.

13.4 Mathematical analogs

The table below identifies mathematical analogs of stress/strain parameters.

Stress/strain	Mathematical analog
ε	x
σ	y
E	(y/x)
$\sigma\{\varepsilon\}$	$y\{x\}$
$E\{\varepsilon\}$	$(y/x)\{x\}$

Note the following in the table:

- The mathematical analog of E is (y/x). Mathematics has no use for (y/x) because (y/x) combines x and y, thereby greatly complicating the solution of nonlinear problems. In mathematics, every effort is made to separate x and y—ie to eliminate terms such as (y/x). Yet the analog of (y/x) is the basis for analysis in modulus methodology.

- The mathematical analog of an $E\{\varepsilon\}$ chart such as Figure (11-1M) is a chart in the form (y/x) vs x. This form is never used in mathematics because it largely conceals the behavior it is intended to reveal. Yet it is the form required in modulus methodology. Note that when this form is used and the value of y is given, an iterative procedure is required *simply to read the graph*—as in Problem 11.7M.

- The mathematical analog of stress/strain behavior $\sigma\{\varepsilon\}$ is $y\{x\}$, the form of choice in mathematics.

In summary, the mathematical analogs in the table above indicate that stress/strain behavior methodology is mathematically desirable, and that stress/strain modulus methodology is mathematically undesirable.

13.5 Why stress/strain modulus is mathematically undesirable.

Stress/strain modulus is mathematically undesirable because it combines the variables σ and ε. Note the following:

- If the variables are combined, proportional problems may be solved in a direct manner, but nonlinear problems must generally be solved in an indirect manner.

- If the variables are separated, proportional problems *and* nonlinear problems may be solved in a direct manner.

- Indirect solutions are more difficult, more time-consuming, and more likely to contain errors than direct solutions.

The variables σ and ε are separated in behavior methodology, and combined in modulus methodology. Therefore nonlinear stress/strain problems may be solved in a direct manner if behavior methodology is used, but must generally be solved in an indirect manner if modulus methodology is used. Because indirect solutions of nonlinear problems are much more difficult than direct solutions, nonlinear stress/strain problems are much easier to solve using behavior methodology.

13.6 The significance of the Problems in Chapters 11 and 12

- Problems 11.5 and 11.5M demonstrate that stress/strain problems that concern only proportional behavior are readily solved using either behavior methodology or modulus methodology.

- Problems 11.6 and 11.6M also demonstrate that stress/strain problems that concern only proportional behavior are readily solved using either behavior methodology or modulus methodology.

- Problems 11.7 and 11.7M demonstrate that stress/strain problems that must be solved in an indirect manner using modulus methodology can be solved in a direct and much simpler manner using behavior methodology. Note that modulus methodology:

    - Requires an iterative procedure simply to read the chart.

    - Requires at least 100 times longer to solve the problem.

    - Increases the likelihood of error by a factor of 100 or more.

Note that if Problem 11.7M concerned the determination of stress at a given strain rather than the determination of strain at a given stress, the solution could be determined in a direct manner using modulus methodology.

However, if Problem 11.7M concerned a *system* of several components including the component in Problem 11.7M, modulus methodology would require an indirect solution whether the problem concerned the determination of stress given strain, or the determination of strain given stress.

- Problems 11.8 and 11.8M also demonstrate that stress/strain problems that must be solved in an indirect manner using modulus methodology can be solved in a direct and much simpler manner using behavior methodology.

  Problem 11.8M is so difficult to solve using modulus methodology that it is unlikely any reader will solve it correctly and completely without using a computer.

  Even if the reader uses a computer, there is a considerable likelihood of error because the reader may not program the computer to find all possible solutions.

  And even if the reader finds all possible solutions, there is a considerable likelihood of error because it may not be readily apparent that one of the solutions is unstable, and is not a steady-state solution. (See Chapter 14.)

13.7 Conclusions

In order to simplify the solution of nonlinear stress/strain problems:

- Modulus and modulus methodology should be abandoned.

- Stress/strain phenomena should be described and analyzed using behavior methodology.

Chapter 14

Irreversible stress/strain behavior

14 Introduction

The stress/strain behavior of metals is generally irreversible in the inelastic region—ie stress is not a unique function of strain—stress also depends on work history. Therefore the relationship between stress and strain is not described by a line on a stress vs strain chart, but rather is described by a two-dimensional zone.

In previous chapters, the subject material was idealized in that it exhibited reversible stress/strain behavior throughout the elastic and inelastic regions. The material was idealized in this way so that the analyses would be closely analogous to analyses of electrical and heat transfer problems in earlier chapters.

In this chapter, the subject material exhibits irreversible stress/strain behavior in the inelastic region. This chapter demonstrates that irreversible stress/strain behavior is dealt with simply and effectively if σ and ε are kept separate—ie if stress/strain behavior methodology is used.

14.1 Reversible and irreversible stress/strain behavior

If the stress/strain behavior of a material is reversible, the following apply:

- Stress is uniquely determined by strain.
- The material is not subject to permanent strain.
- The relationship between stress and strain is one-dimensional—ie is described by a single line on a stress vs strain chart.

If the stress/strain behavior of a material includes an irreversible region, the following apply:

- Stress is *not* uniquely determined by strain. Stress also depends on work history.

- The material is subject to permanent strain.

- The relationship between stress and strain is two-dimensional—ie is described by an area on a stress vs strain chart.

In the elastic region, stress/strain behavior is reversible. In the inelastic region, the stress/strain behavior of metals is usually irreversible, and results in permanent strain.

14.2 A definition of “stress/strain diagram”

The following is a reasonable definition of “stress/strain diagram”:

> *The stress/strain diagram is the locus of points that describe the relationship between stress and strain.*

Figure (14-1) is a stress/strain diagram of a more or less typical material in its virgin state—ie with no initial permanent strain. The solid curve in Figure (14-1) would result if the strain were monotonically increased from zero strain to fracture.

Each dashed line indicates how stress would vary if the strain was monotonically increased to the upper limit of the dashed line, and the load was then decreased to zero. Since the dashed lines do not coincide with the solid line, the stress/strain behavior is irreversible, and the result is permanent strain.

The myriad of dashed lines in Figure (14-1) indicates that, when the stress/strain behavior is irreversible, the stress/strain diagram is essentially the area under the curve.

Figure 14-1 **Stress/strain diagram**

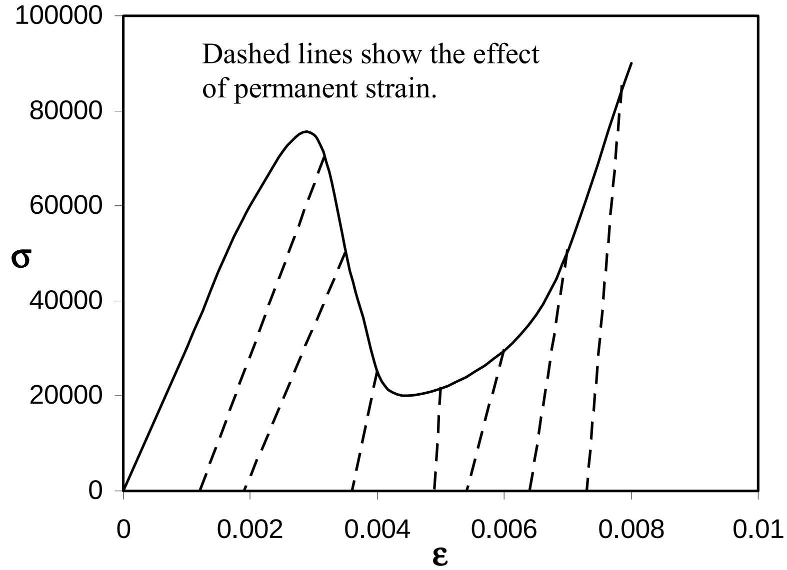
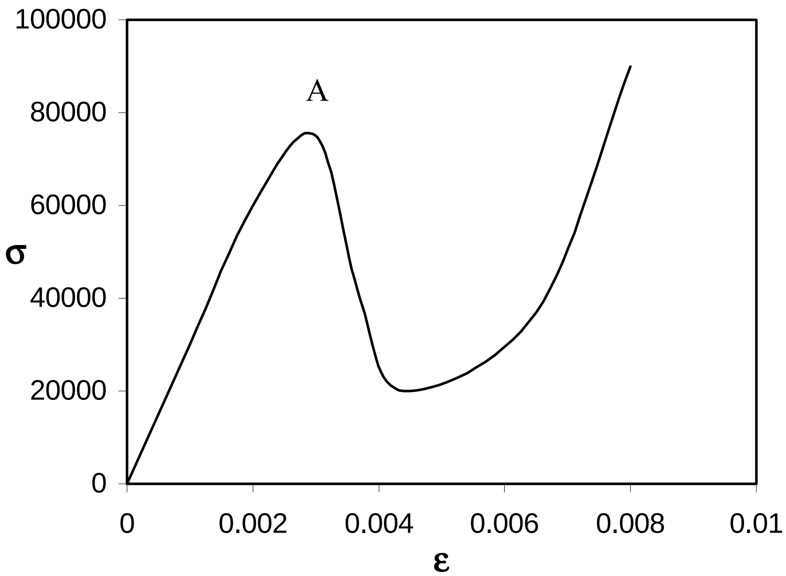


Figure 14-2 **Stress/strain curve based on Figure 14-1**



14.3 A definition of "stress/strain curve"

The following is a reasonable definition of "stress/strain curve":

> *The stress/strain curve is the upper boundary of the stress/strain diagram.*

Figure (14-2) is the stress/strain curve for the virgin material described in Figure (14-1).

14.4 Measuring the stress/strain curve

Stress/strain data can be obtained using apparatus in which load is the controlled variable, or strain is the controlled variable. However, if the stress/strain curve contains a maximum, the curve can be measured in its entirety only if strain is the controlled variable.

For example, note in Figure (14-2) that, if the *load* is slowly and monotonically increased from zero to fracture, a step increase in strain results when the stress passes through the maximum at Point A—ie there is a step from (.0029, 75000) to (.0076, 75000). Thus the test data will give the result in Figure (14-3) rather than the desired result in Figure (14-2).

If the load is reversed just before the fracture point is reached, and if the load is then monotonically reduced to zero, the test data will give the result shown in Figure (14-4) rather than the desired result shown in Figure (14-2).

On the other hand, note in Figure (14-2) that, if the *strain* is monotonically increased from zero strain to fracture, the stress/strain curve will be measured in its entirety because stress is a single-valued function of strain. Therefore the test data will give the result shown in Figure (14-2), as desired.

In summary:

- If a strain-controlled apparatus is used, the stress/strain curve can be measured in its entirety, even if $\sigma\{\varepsilon\}$ contains a maximum.
- If a load-controlled apparatus is used, the stress/strain curve can be measured in its entirety *only* if $\sigma\{\varepsilon\}$ does not contain a maximum—ie only if stress increases monotonically with strain from zero strain to fracture.

Figure 14-3 **Load increased monotonically, stress/strain curve from Figure 14-2**

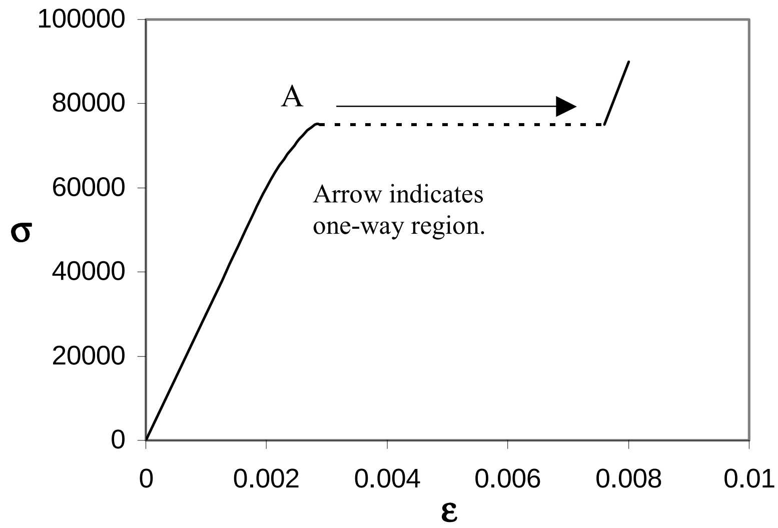
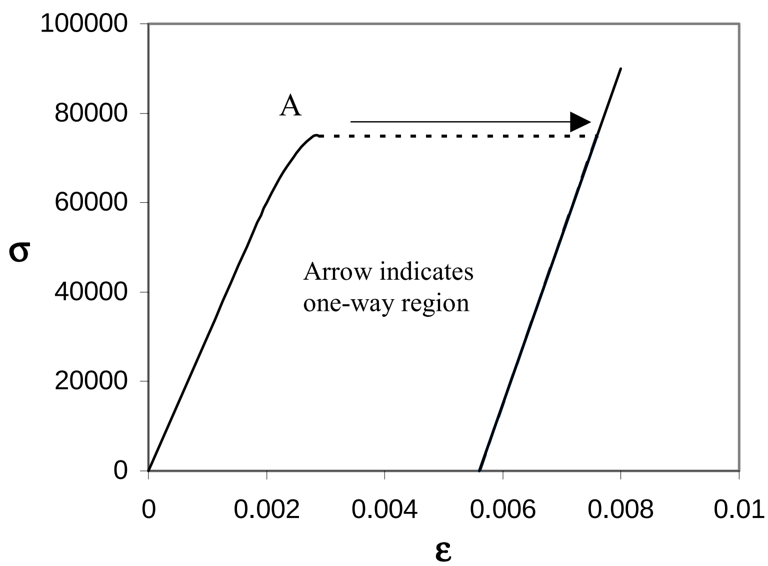


Figure 14-4 **Load in Figure 14-3 decreased monotonically from σ = 90000 to 0**



14.5 Summary of stress/strain diagram and stress/strain curve

- The stress/strain diagram is the locus of points that describe the relationship between σ and ε.

- The stress/strain curve is the upper boundary of the stress/strain diagram.

- In the elastic region:
  - Operation is reversible.
  - Permanent strain does not result.
  - The stress/strain diagram is one-dimensional.

- In the inelastic region:
  - Operation is generally irreversible.
  - Permanent strain generally results.
  - The stress strain diagram is generally two-dimensional.

- In order to uniquely determine the stress and strain in a material that is subject to permanent strain, it is necessary to know the work history of the material in order to modify the stress/strain diagram of the virgin material to account for the effect of work history.

- If the stress/strain *curve* of the virgin material is used to analyze a material that is subject to permanent strain, the calculated value of strain will be the minimum that could result, since permanent strain increases strain. The calculated value of stress will be the maximum stress possible, since increased strain usually corresponds to smaller applied load.

14.6 Impact of permanent strain on Problems 11.5 and 11.6

Permanent strain has no impact on the solution of Problems 11.5 and 11.6. Those problems concern elastic behavior, and permanent strain is an inelastic phenomenon.

14.7 Impact of permanent strain on Problem 11.7

Problem 11-7 concerns the strain that would result from a stress of 40,000 lbs in an idealized material not subject to permanent strain. The complete solution is .0013, .0037, or .0066.

In order to revise Problem (11-7) so that it concerns a material that is subject to permanent strain, revise the problem in the following ways:

- State that the stress/strain diagram of the material in its virgin state is described by Figure (14-1) rather than Figure (11-1).
- State that the work history of the material is unknown.

Because the work history of the material is unknown, the amount of permanent strain is unknown. Therefore the solution of the problem must allow for the potential effect of permanent strain. The net result is that a specific value of stress can result in a wide range of strain values.

The solution is obtained by inspecting Figure (14-1) to determine the intersections between a line drawn at stress = 40,000 psi and the myriad of dashed lines that describe behavior in a permanently strained condition. The complete solution of the revised problem is:

> The strain may be any value between .0013 and .0037, and any value between .0066 and .0075.

In order to uniquely determine the strain that would result from a stress of 40,000 psi, it would be necessary to know the work history.

14.8 Impact of permanent strain on Problem 11.8—work history unknown

Problem (11.8) involves a bar material that is not subject to permanent strain. The complete solution obtained from Figure (11-2) is:

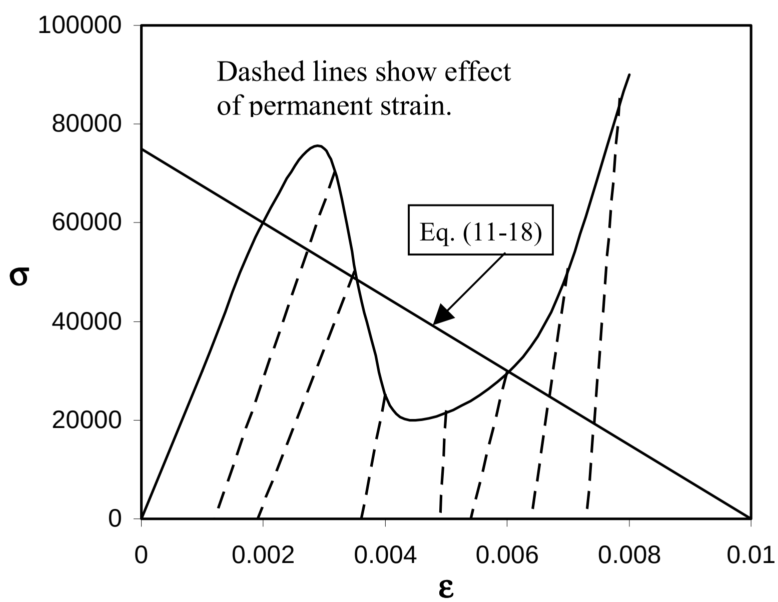
Stress, psi	**Strain**
60,000	.0020
49,000	.0035
30,000	.0060

In order to revise Problem (11-8) so that it concerns a material that is subject to permanent strain and has unknown work history, revise the problem in the following ways:

- State that the stress/strain diagram of the material in its virgin state is described by Figure (14-1) rather than Figure (11-1).
- State that the work history of the material is unknown.
- Generate Figure (14-5) by plotting Eq. (11-18) on Figure (14-1).

The solution is obtained by inspecting Figure (14-5) to determine the intersections that occur between Eq. (11-18) and the myriad of dashed lines that describe behavior in the permanently strained condition.

Figure 14-5 **Solution of problem in Section 14.8, stress/strain diagram from Figure 14-1.**



The complete solution of the revised problem is:

> The stress may be any value between 49,000 and 60,000 psi, *and* any value between 20,000 and 30,000 psi. The strain may be any value between.002 and .0035, *and* between .006 and .0075.

14.9 Impact of permanent strain on Problem 11.8—work history known

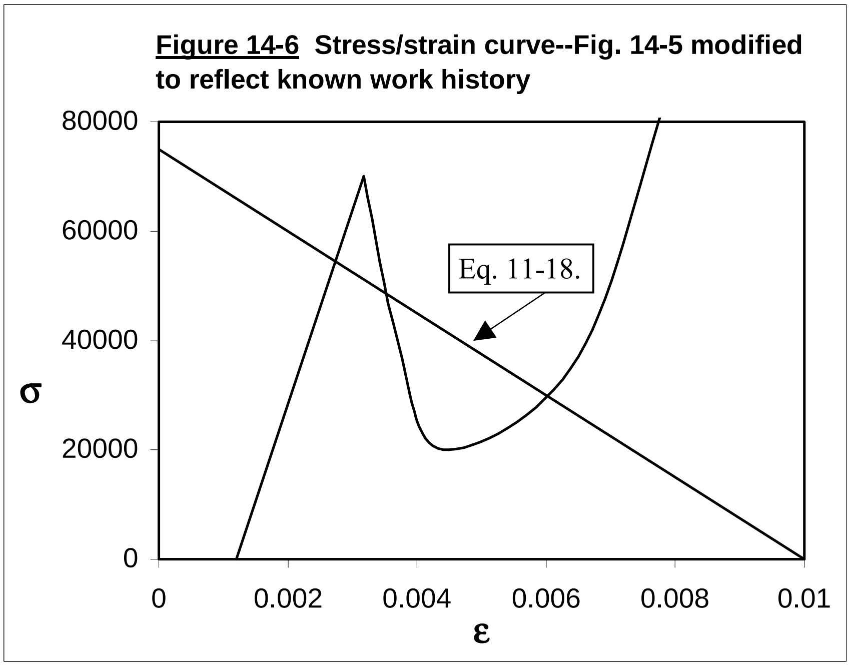
In order to revise Problem (11-8) so that it concerns a material that is subject to permanent strain and has a known work history, revise the problem in the following ways:

- Revise the given information to state that the stress/strain diagram of the material in its virgin state is described by Figure (14-1) rather than Figure (11-1).
- Revise the given information to state that the work history of the material indicates a lifetime maximum strain of .0032.
- Redraw Figure (14-4) to reflect the work history of the material:
  - Delete the stress/strain curve at strain values less than .0032.
  - Change the dashed line that intersects the stress/strain curve at ε = .0032 to a solid line.
  - Delete the other dashed lines, and note that the result is a stress/strain curve. (The stress/strain *curve* is appropriate because there is no ambiguity about the condition of the material, since the work history is known.).
- Plot Eq. (11-18) on Figure (14-6).

The solution is obtained by inspecting Figure (14-6). The complete solution of the revised problem is:

The stress is 55,000 psi and the strain is .0027.

Note that the only solution is in the elastic region. The other two intersections in Figure (14-6) are not solutions because they are at strain values larger than .0032, and the work history indicates that the strain never exceeded .0032.



14.10 Stability at potential stress/strain operating points

Stability at potential stress/strain operating points is dealt with in the following manner:

- If a stress/strain analysis indicates a triple-valued solution, the middle intersection is unstable and is ignored. If the system somehow arrives at the middle intersection, it simply refuses to remain there, and operation shifts to one of the other two potential operating points. (If the material were subject to permanent strain, operation would not shift to the solution at the lower strain.)

- Undamped oscillations do *not* result from unstable operating points because stress/strain curves do not exhibit the type of behavior required to generate undamped oscillations.

14.11 Conclusions

Irreversible stress/strain behavior is dealt with simply and effectively if σ and ε are kept separate—ie if stress/strain behavior methodology is used.

Chapter 15

A critical examination of fluid friction factor

15 Introduction

This chapter critically examines the concept/parameter known as fluid "friction factor". The examination reveals that "friction factor" is a parameter group that combines the primary parameters flow rate W and pressure drop ΔP, thereby making it necessary to solve fluid flow problems with the variables *combined*.

Since it is generally easier to solve problems with the variables separated, it is concluded that "friction factor" should be abandoned, and should be replaced by behavior methodology—ie by methodology that allows fluid flow problems to be solved with W and ΔP *separated*.

15.1 Problems to be solved by readers who are familiar with Moody charts

1. Moody charts are in the form friction factor versus Reynolds number with relative roughness as a parameter. Describe how to read a Moody chart to find the friction factor if the given information is pressure drop, fluid properties, relative roughness, and geometry.

2. On Figure (15-1), sketch $\Delta P\{W\}$ for steady, incompressible flow in a smooth, constant area duct. (Notice that Figure (15-1) is a *linear* chart.) In other words, *qualitatively* describe how ΔP increases as W is increased from zero. A Moody chart may be used for reference.

15.2 "Friction factors"

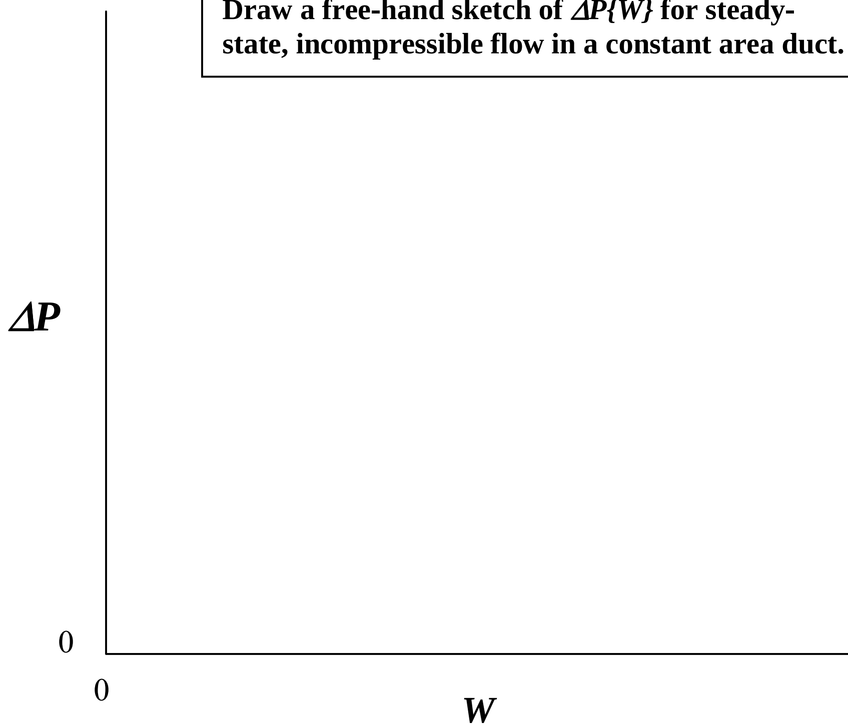
There are two fluid flow "friction factors"—the Darcy friction factor and the Fanning friction factor. They are defined as follows:

The Darcy friction factor is $(2gD\rho A^2\Delta P/LW^2)$.

The Fanning friction factor is $(gD\rho A^2\Delta P/2LW^2)$.

<u>Figure 15-1</u> ***To be completed by the reader.***

Draw a free-hand sketch of $\Delta P\{W\}$ for steady-state, incompressible flow in a constant area duct.



Notice that both friction factors combine ΔP and W in parameter groups that are proportional to $\Delta P/W^2$. Therefore ΔP and W can be separated only if friction factor is *eliminated*.

It is generally agreed that friction factor is related to Reynolds number and relative wall roughness in the following ways:

- In laminar flow, the friction factor is a function of Reynolds number $(D/\mu A)W$. The Darcy laminar friction factor is $64/N_{Re}$. The Fanning laminar friction factor is $16/N_{Re}$.

- In turbulent flow, the friction factor is a function of Reynolds number and wall relative roughness ε/D.

- If the flow is turbulent and the wall relative roughness is very large, the friction factor depends primarily on wall relative roughness, and little depends on Reynolds number.

The Moody chart is in the form friction factor vs Reynolds number, with wall relative roughness as parameter. Charts such as the Moody chart are used to describe the relationship between flow rate, pressure drop, wall relative roughness, fluid properties, and geometry.

15.3 The Moody chart as described by Moody

Moody (1944) describes the purpose of the Moody chart:

> *The author does not claim to offer anything particularly new or original, his aim merely being to embody the now accepted conclusions in convenient form for engineering use.*

Moody explains why the coordinates in the Moody chart are friction factor and Reynolds number, since other coordinates were also in use in 1944:

> *. . . R. J. S. Pigott (1933) published a chart for the (Darcy friction factor), using the same coordinates (used in the Moody chart). His chart has proved to be most useful and practical and has been reproduced in a number of texts.*

Moody divided the chart into four Reynolds number zones, and used literature correlations and generally accepted views to generate the curves in each zone. The following describes the four zones and the manner in which Moody determined the relationship between friction factor and Reynolds number in each zone:

- Laminar flow zone: Reynolds number is less than 2000. Line in chart is Hagen-Poiseuille law, $f_{Darcy} = 64/N_{Re}$.

- Critical zone: Reynolds number is 2000 to 4000. Friction factor is an indeterminate value between laminar flow value and turbulent flow value. Zone is shown as a gray area.

- Transition zone: Region between the smooth wall curve and the lower limit of the rough-pipe zone. The smooth wall curve was determined from an equation attributed to von Karman (1930), Prandtl (1933), and Nikuradse (1933). The curves within the transition region were determined from the Colebrook (1938-1939) function. The lower limit of the rough-pipe zone was determined from an equation used by Rouse (1943) to generate a friction factor chart.

- Rough-pipe zone: Lower limit of zone was determined from an equation used by Rouse (1943) to generate a friction factor chart. Lines within the zone were determined from the generally accepted view that, in rough-pipe zone, friction factor is independent of Reynolds number. Zone has no upper limit.

With regard to the accuracy of the Moody chart, Moody states:

> *It must be recognized that any high degree of accuracy in determining the friction factor is not to be expected.*

15.4 Why "friction factor" is undesirable

"Friction factor" is the parameter group $(gD\rho A^2\Delta P/LW^2)$ multiplied by 2 (to give the Darcy friction factor) or divided by 2 (to give the Fanning friction factor). Both friction factors are undesirable because they combine the primary variables W and ΔP, thereby making it necessary to solve fluid flow problems with W and ΔP combined, even though it is generally much simpler to solve nonlinear problems if the variables are separated.

15.5 Mathematical analogs of fluid flow parameters

The primary parameters in fluid flow engineering are flow rate W and pressure drop ΔP. The mathematical analogs of fluid flow parameters are listed below:

Parameter	Symbolic form	Mathematical Analog
Fluid flow rate, pps	W	x
Pressure drop, psi	ΔP	y
Reynolds No.	$(D/\mu A)W$	ax
Fanning friction factor	$(gD\rho A^2/2L)(\Delta P/W^2)$	$b(y/x^2)$
Darcy friction factor	$(2gD\rho A^2/L)(\Delta P/W^2)$	$4b(y/x^2)$

15.6 Why the Moody chart is undesirable

In the Moody chart, the relationship between W and ΔP is described in the form friction factor $(2gD\rho A^2/L)(\Delta P/W^2)$ vs Reynolds number $(D/\mu A)W$. This form is undesirable because:

- If ΔP is given and the problem is to determine W, neither the Reynolds number nor the friction factor can be calculated directly from the given information. Consequently an indirect procedure is required *simply to read the Moody chart*. The Moody chart can be read in a direct manner *only* if W is included in the given information.

- The Moody chart is in the form $a\Delta P/W^2$ vs bW. This form is undesirable because it largely conceals the relationship it is intended to describe—namely, the relationship between ΔP and W.

- The mathematical analog of the Moody chart is a chart of y/x^2 vs x. In pure mathematics, it would be unheard of to describe a highly nonlinear function such as $\Delta P\{W\}$ with a chart in the form y/x^2 vs x. Note that $\Delta P\{W\}$ includes a region in which ΔP is proportional to W, a region in which there is a step change in $\Delta P\{W\}$, and a region in which ΔP is proportional to W raised to a power between 1.8 and 2.

In summary, the Moody chart is undesirable because it is in an inconvenient form—so inconvenient that it must be read in an indirect manner if ΔP is given, and W is to be determined. Yet the Moody chart is widely used in conventional engineering because it is, in Moody's words, *in convenient form for engineering use*.

15.7 The purpose of Problem 1

The purpose of Problem 1 is to illustrate that the Moody chart must be read in an indirect and undesirable manner if W is not given. Note that both friction factor and Reynolds number are functions of W, and therefore the value on *neither* axis can be calculated if W is not given. Therefore if ΔP is given and the problem is to determine W, the Moody chart must be read in an indirect manner such as the following:

- Estimate the flow rate.
- Calculate a Reynolds number based on the estimated flow rate.
- Read the Moody chart to determine the friction factor at the above Reynolds number and the given relative roughness.
- Use the above friction factor and the given ΔP to determine a better estimate of the flow rate.
- Repeat the above until the solution converges.
- If the solution diverges, use a different indirect method.

It is important to note that the reason an indirect method is required to read the Moody chart to determine W is because W and ΔP are combined in friction factor. If the Moody chart is transformed to eliminate friction factor, the resultant chart can be read directly if W is given and ΔP is to be determined, and also if ΔP is given and W is to be determined.

15.8 The purpose of Problem 2

The purpose of Problem 2 is to illustrate that the Moody chart largely conceals the relationship it is intended to describe—namely, the relationship between ΔP and W. The problem requires that the implicit and quantitative description of $\Delta P\{W\}$ given in the Moody chart be transformed to an explicit and qualitative description of $\Delta P\{W\}$. Figure (16-2) presents the solution to Problem 2.

A chart in the form y/x^2 vs x would *not* be used in mathematics to describe a $y\{x\}$ relationship similar to $\Delta P\{W\}$ because the chart would largely conceal the relationship between x and y. Yet y/x^2 vs x is the mathematical analog of the Moody chart, and the Moody chart is widely used in conventional engineering.

15.9 Friction factor in laminar flow

In conventional engineering, Eq. (15-1) describes the relationship between fluid flow rate and pressure drop. It is used for both laminar flow and turbulent flow.

$$\Delta P_{laminar\ or\ turbulent} = f_{Darcy}\,(L/2gD\rho A^2)(W^2) \qquad (15\text{-}1)$$

Consider the following:

- Eq. (15-1) *appears* to state that $\Delta P_{laminar}$ is proportional to W^2, since it *explicitly* states that $\Delta P_{laminar}$ is proportional to W^2.
- Eq. (15-1) does *not* state that $\Delta P_{laminar}$ is proportional to W^2.
- The discrepancy between appearance and reality results because Eq. (15-1) describes the relationship between ΔP and W both implicitly and explicitly.
- The relationship between ΔP and W is in part described implicitly, since $f_{laminar}$ is a function of W. (Recall that, in laminar flow, $f_{Darcy} = 64/N_{Re} = 64/(DW/\mu A)$.)
- When both explicit and implicit functionalities are considered, Eq. (15-1) actually states that $\Delta P_{laminar}$ is proportional to W, even though it appears to state that $\Delta P_{laminar}$ is proportional to W^2.

In order to make appearance agree with reality, f_{Darcy} must be eliminated from Eq. (15-1). This is accomplished by substituting $64/(DW/\mu A)$ for $f_{Darcy,laminar}$, resulting in Eq. (15-2). Note that Eq. (15-2) explicitly and correctly indicates that $\Delta P_{laminar}$ is proportional to W. In other words, in Eq. (15-2), appearance agrees with reality, as desired.

$$\Delta P_{laminar} = (32\mu L/gD^2\rho A)W \qquad (15\text{-}2)$$

Also note that ΔP and W are separate and explicit in Eq. (15-2). Therefore Eq. (15-2) is a behavior equation, and is in the form required in the new engineering.

Eq. (15-2) is often used in conventional engineering in place of Eq. (15-1). However, Eq. (15-2) is so superior to Eq. (15-1) that it is surprising that Eq. (15-1) and friction factor are *ever* used to describe or analyze laminar flow.

15.10 Summary

- Friction factor is the dimensionless group $(gD\rho A^2\Delta P/LW^2)$ multiplied or divided by 2.

- Friction factor is undesirable because it combines ΔP and W, thereby making it necessary to solve fluid flow problems with the variables combined, even though nonlinear problems are generally much easier to solve if the variables are separated.

- The Moody chart is undesirable because it is in the form friction factor vs Reynolds number. Since the parameters on both axes are functions of W, the chart must be read in an indirect manner when ΔP is given, and W is sought.

- The Moody chart is undesirable because it is in a form that largely conceals the relationship between ΔP and W, the relationship it is intended to reveal.

- Even in conventional engineering, friction factor should not be used to describe or analyze laminar flow because it results in an equation that explicitly states that ΔP is proportional to W^2, when in fact it states that ΔP is proportional to W.

Chapter 16

Fluid flow behavior methodology

16 Introduction

Chapter 15 demonstrates that "friction factor" and friction factor charts such as the Moody chart do not provide a desirable methodology for dealing with fluid flow. The methodology is undesirable because friction factor combines the primary parameters ΔP and W, thereby making it necessary to solve problems with ΔP and W combined, even though nonlinear problems are generally much easier to solve if the variables are separated.

This chapter presents fluid flow methodology based on fluid flow "behavior"—ie methodology in which ΔP and W are *separate* and *explicit*. Friction factor and the Moody chart are abandoned.

16.1 The behavior replacements for "friction factor" and the Moody chart

The Moody chart presents fluid flow information in a form that is inconvenient for engineering use—friction factor $(2Dg\rho A^2/L)(\Delta P/W^2)$ vs Reynolds number $(D/\mu A)W$. The inconvenience results because ΔP and W are combined in friction factor.

In order to present the information in the Moody chart in a more convenient form, the chart is transformed to the behavior form. This eliminates friction factor, and separates ΔP and W.

The Moody chart is transformed to behavior form in the following way:

- Note that the Darcy friction factor is $(2Dg\rho A^2/L)(\Delta P/W^2)$ and that the Reynolds number is $(D/\mu A)W$.

- Note that in order to separate ΔP and W, a parameter group is required that is a function of ΔP, but is independent of W.

- Note that if f_{Darcy} is multiplied by $0.5N_{Re}^2$, the resultant parameter group is $(D^3g\rho/L\mu^2)\Delta P$. (If the Moody chart is in terms of $f_{Fanning}$, the multiplier is $2N_{Re}^2$.) This parameter group is a function of ΔP, but is independent of W, as desired

- Note that if $(D^3g\rho/L\mu^2)\Delta P$ is plotted vs $(D/\mu A)W$, and if the axes titles are in explicit form, the resultant chart will be in behavior form—ie ΔP and W will be *separate* and *explicit*.

In summary:

- In behavior methodology, the dimensionless parameters called "Darcy friction factor" and "Fanning friction factor" are replaced by the dimensionless parameter $(D^3g\rho/L\mu^2)\Delta P$. This parameter is used in explicit form, and is *not* assigned a name such as "Smith number" because it includes one of the primary parameters.

- In behavior methodology, the Moody chart is replaced by the fluid flow "behavior" chart—a chart of $(D^3g\rho/L\mu^2)\Delta P$ vs $(D/\mu A)W$, with ε/D parameter. The axes titles are in explicit form:

  - The x-axis is entitled "$(D/\mu A)W$" rather than Reynolds number.

  - The y-axis is entitled "$(D^3g\rho/L\mu^2)\Delta P$" rather than an assigned name such as Smith number.

- Because ΔP and W are separated in fluid flow behavior charts, the charts can be read directly if W is given and ΔP is sought, *and* if ΔP is given and W is sought.

16.2 Generating the fluid flow behavior chart

The fluid flow behavior chart is generated by transforming the Moody chart from f_{Darcy} (or $f_{Fanning}$) vs Reynolds number to $(D^3g\rho/L\mu^2)\Delta P$ vs $(D/\mu A)W$. The transformation is accomplished as follows:

- On a spreadsheet, list (f_{Darcy},N_{Re}) or $(f_{Fanning},N_{Re})$ coordinates for each curve in the Moody chart.

- Multiply the f_{Darcy} coordinates by $0.5N_{Re}^2$ (or the $f_{Fanning}$ coordinates by $2N_{Re}^2$), resulting in coordinates of $((D^3g\rho/L\mu^2)\Delta P, (D/\mu A)W)$.

- Plot $(D^3 g\rho/L\mu^2)\Delta P$ vs $(D/\mu A)W$.
- Entitle the y-axis "$(D^3 g\rho /L\mu^2)\Delta P$", and the x-axis "$(D/\mu A)W$".
- Entitle the chart "Fluid flow behavior chart".

Figure (16-1) is a fluid flow behavior chart. It was obtained by transforming Moody chart curves for a smooth wall, and for a wall of relative roughness $\varepsilon/D = .003$. Notice that Figure (16-1) is on two pages, and that it is incomplete in that it omits many of the ε/D values found in the Moody chart.

16.3 Advantages of fluid flow behavior charts relative to friction factor charts

Fluid flow behavior charts such as Figure (16-1) have the following advantages relative to friction factor charts such as the Moody chart.

- Fluid flow behavior charts can be read directly if ΔP is given and W is sought, *and* if W is given and ΔP is sought. Friction factor charts such as the Moody chart can be read directly if W is given and ΔP is sought, but must be read in an indirect manner if ΔP is given and W is sought.
- Fluid flow behavior charts readily reveal the qualitative relationship between ΔP and W because they are in the form $a\Delta P$ vs bW. Friction factor charts largely conceal the relationship between ΔP and W because they are in the form $c\Delta P/W^2$ vs bW.

Recall that Moody's stated purpose in preparing the Moody chart was to put accepted knowledge of fluid flow behavior *"in convenient form for engineering use"*, and that he used the same coordinates used by Pigott because Pigott's *"chart has proved to be most useful and practical . . . "*.

On the bases of convenience, usefulness, and practicality, the fluid flow behavior chart should replace the Moody chart because the Moody chart can be read directly in only one direction, whereas the fluid flow behavior chart can be read directly in both directions.

Figure 16-1A **Fluid flow behavior chart**

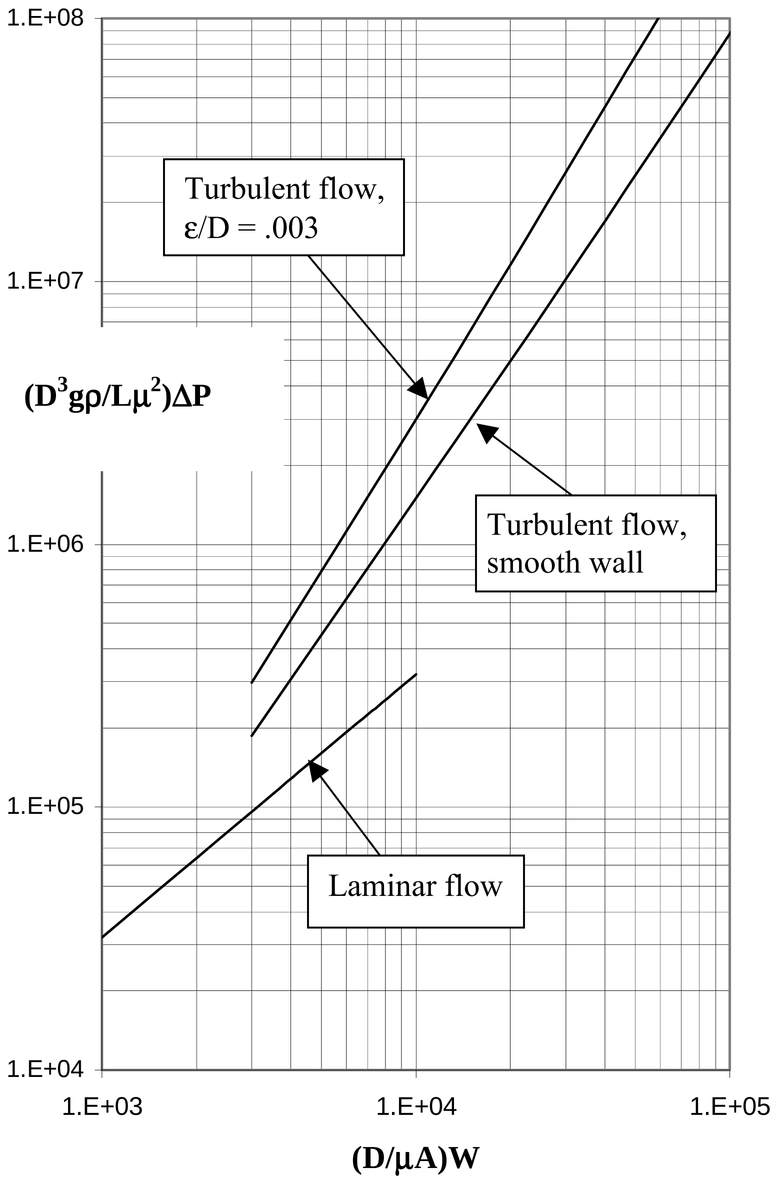
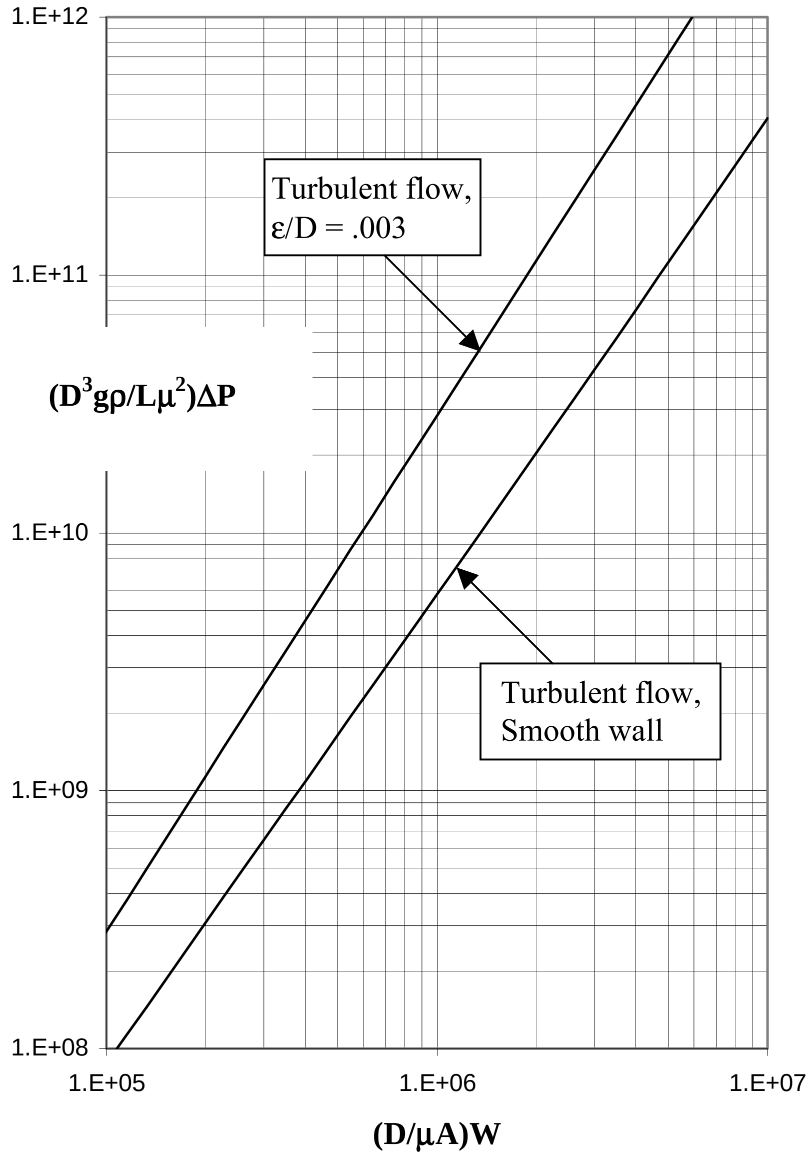


Figure 16-1B Fluid flow behavior chart



16.4 Parameter groups in fluid flow behavior methodology

Recall from an earlier chapter that, in the new engineering:

- Parameter groups that include both primary parameters are *not* used because combining the primary parameters is mathematically undesirable.

- Parameter groups that include one primary parameter are used *only* in explicit form in order that the primary parameter will appear explicitly.

- Parameter groups that do not include a primary parameter are used in both explicit and implicit form.

Therefore, in fluid flow behavior methodology:

- Friction factor is *not* used because it combines ΔP and W.

- $(D^3 g\rho/L\mu^2)\Delta P$ is used *only* in explicit form because it includes ΔP. It is *not* assigned a name or a symbol.

- Reynolds number is used *only* in explicit form because it includes W. In other words, $(DW/\mu A)$, (DG/μ), and $(DV\rho/\mu)$ are used, but Reynolds number and N_{Re} are *not*.

16.5 Solution of Problem 2 in Chapter 15

The solution of Problem 2 in Chapter 15 is readily apparent by noting that the fluid flow behavior chart, Figure (16-1A), is a quantitative description of $\Delta P\{W\}$ in logarithmic form. The logarithmic form is transformed to linear form by noting the following in Figure (16-1A):

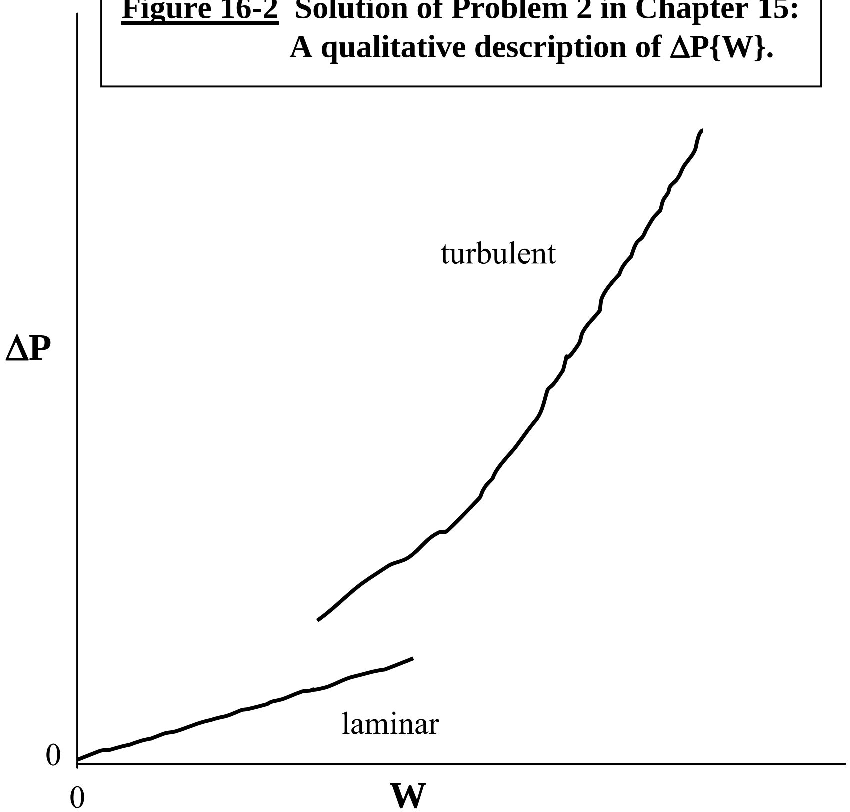
- All the curves in the figure are essentially straight lines, and therefore the curves are essentially of the form $\Delta P = mW^n$.

- The exponent in the laminar region is one. (Note that when W increases a factor of 10, ΔP increases a factor of 10, indicating that the exponent is 1.) Therefore the laminar flow region is described by a straight line that starts at the origin.

- The exponent in the turbulent region is approximately 2. (Note that when W increases a factor of 10, ΔP increases a factor of approximately 100, indicating that the exponent is approximately 2.) Therefore the turbulent region is described by an exponential curve. The turbulent line begins near the upper end of the laminar region, and above the laminar line.

The solution of Problem 2 in Chapter 15 is shown in Figure (16-2).

Figure 16-2 **Solution of Problem 2 in Chapter 15: A qualitative description of ΔP{W}.**



16.6 Simple, analytical expressions for fluid flow behavior

In most practical cases, there is no need to use the fluid flow behavior chart because the curves in the chart are accurately described by simple, easy-to-use equations. In the laminar region, fluid flow behavior is described by Eq. (15-2). In the turbulent region, fluid flow behavior is described by equations in the form of Eq. (16-1) where m and n are constants whose value depends solely on the value of ε/D.

$$(D^3 g\rho/L\mu^2)\Delta P_{turb} = m(DW/\mu A)^n \qquad (16\text{-}1)$$

For example, for smooth wall tubes, $\Delta P\{W\}$ determined from Eq. (16-2) closely agrees with $\Delta P\{W\}$ determined from the Moody chart. For relative wall roughness of .003, $\Delta P\{W\}$ determined from Eq. (16-3) closely agrees with $\Delta P\{W\}$ determined from the Moody chart.

$$(D^3 g\rho/L\mu^2)\Delta P_{turb,smooth} = 0.0755\ (DW/\mu A)^{1.8142} \qquad (16\text{-}2)$$

$$(D^3 g\rho/L\mu^2)\Delta P_{turb,e/D\,=\,.003} = 0.0289\ (DW/\mu A)^2 \qquad (16\text{-}3)$$

The close agreement between chart and equations is shown in Figure (16-3). Note that the solid lines obtained by transforming Moody chart curves are barely distinguishable from the dashed lines based on Eqs. (16-2) and (16-3). And recall Moody's statement:

> *It must be recognized that any high degree of accuracy in determining the friction factor is not to be expected.*

16.7 Analytical expressions for turbulent friction factor

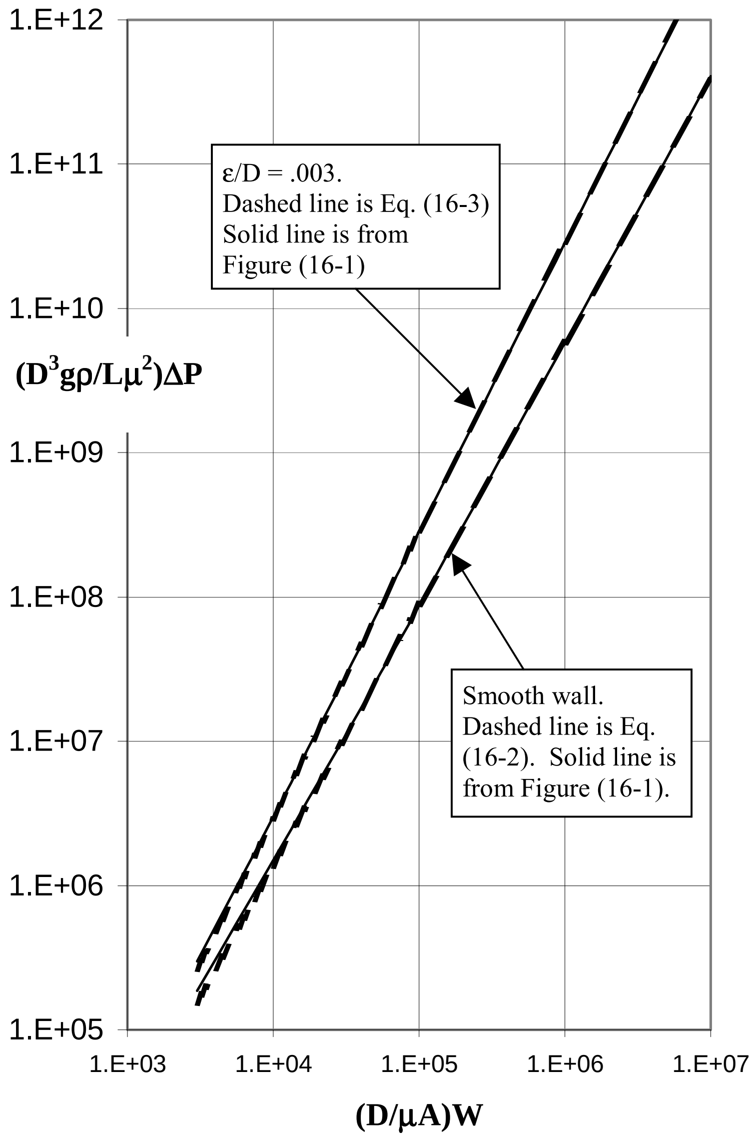
The friction factor curves in the Moody chart may also be described analytically, but the equations for turbulent flow are not easy to use. Moody states that the smooth wall curve was obtained from Eq. (16-4), and the curves in the transition region were obtained from Eq. (16-5).

$$1/(f_{Darcy})^{.5} = 2\ \log_{10}(N_{Re}\, f_{Darcy}^{\ 0.5}/2.51) \qquad (16\text{-}4)$$

$$1/(f_{Darcy})^{.5} = -2\ \log((\varepsilon/3.7D) + 2.51/(N_{Re}\, f_{Darcy}^{\ 0.5})) \qquad (16\text{-}5)$$

Note that neither equation is easy-to-use.

Figure 16-3 Fluid flow behavior chart-- Comparison with analytical expressions



16.8 A digital form of the fluid flow behavior chart

When Moody's article was published, charts were more convenient than computation, as evidenced by the following charts in Moody's article:

- Figure 1 is the Moody chart. It is used to determine friction factor given any of the following sets:
  - ε/D and Reynolds number.
  - ε/D, the product of velocity in ft/sec and pipe diameter in inches, fluid is water at 60 F.
  - ε/D, the product of velocity in ft/sec and pipe diameter in inches, fluid is air at 60 F.
- Figure 2 is a chart to determine ε/D, given the pipe diameter and either the value of ε or the pipe material.
- Figure 3 is a chart to determine Reynolds number for water at 60 F, given the velocity in ft/sec and the pipe diameter in feet or inches.
- Figure 4 is a chart to determine Reynolds number, given the fluid, the fluid temperature, and the product of velocity in ft/sec and pipe diameter in inches.

Today, because of the widespread use of calculators and computers, simple analytical expressions are much more convenient than charts, and charts are avoided if possible.

The close agreement between the Moody chart curves and the equations in Figure (16-3) suggests that the fluid flow behavior chart should also be presented in digital form. For example, a digital form could be a table that lists ε/D values and the corresponding values of m and n to be used in Eq. (16-1), such as the abbreviated table below.

Digital form of fluid flow behavior chart

ε/D	**m in Eq. (16-1)**	**n in Eq. (16-1)**
smooth	0.151	1.8142
.003	0.578	2

The complete table would:

- Include all ε/D values in the Moody chart.
- State that laminar flow is described by Eq. (15-2).
- State that the transition from laminar flow to turbulent flow usually occurs at (DW/μA) values in the range 2000 to 4000.
- Include a small, linear chart that qualitatively describes ΔP{W} for (DW/μA) values in the range 0 to 10000.

16.9 Summary

Fluid flow behavior methodology is described by the following:

- Fluid flow phenomena are described and problems are solved with W and ΔP separate and explicit. In order for W and ΔP to be separate, parameter groups that combine them are abandoned—ie parameter groups such as friction factor are abandoned
- The group $(D^3 g\rho/L\mu^2)\Delta P$ replaces both f_{Darcy} and $f_{Fanning}$. This ends the ambiguity and confusion caused by the two different friction factors in conventional engineering.
- Fluid flow behavior charts replace Moody charts—ie $(D^3 g\rho/L\mu^2)\Delta P$ vs $(D/\mu A)W$ replaces f_{Darcy} (or $f_{Fanning}$) vs N_{Re}. Fluid flow behavior charts can be read in a direct manner if ΔP is given or if W is given, but Moody charts can be read in a direct manner only if W is given.
- Friction factor methodology is replaced by fluid flow behavior methodology This simplifies the solution of nonlinear fluid flow problems because it allows the primary parameters to be separate, whereas the primary parameters are combined in friction factor methodology.
- In the interest of engineering convenience, the fluid flow behavior chart is also presented in the digital form described in Section 16.8.

Chapter 17

A critical appraisal of the conventional view of dimensional homogeneity

17 Introduction

Fourier conceived the conventional view of dimensional homogeneity—the view that scientific rigor requires engineering phenomena to be described by dimensionally homogeneous equations. Fourier's view resulted in the need to create parameters such as *R*, *h*, and *E* so that engineering phenomena could be described by homogeneous equations.

In this chapter, the conventional view of dimensional homogeneity is described and critically appraised. It is concluded that the conventional view should be abandoned principally because:

- Engineering phenomena generally exhibit *in*homogeneous behavior. Therefore there is no reason to suppose that engineering phenomena are rigorously described only by equations that are homogeneous.

- The conventional view requires the creation of parameters such as *R, h, E* that combine the primary parameters, and greatly complicate the solution of nonlinear problems.

17.1 The role of dimensional homogeneity

Dimensional homogeneity concerns numbers, dimensions, dimensioned quantities, and equations.

- Numbers answer the question "How many?"

- Dimensions answer the question "Of what things?"

- Dimensioned quantities answer the question "How many of what things?"

Dimensional homogeneity dictates which mathematical operations may be performed on numbers, which may be performed on dimensions, and which may be performed on dimensioned quantities. It also dictates what is indicated by the equal sign.

Numbers may obviously be added, subtracted, multiplied, and divided. Therefore the role of dimensional homogeneity is to answer three questions:

- Which mathematical operations may be performed on dimensions?
- Which mathematical operations may be performed on dimensioned quantities?
- What does the equal sign indicate? Numerical equality? Or numerical equality *and* dimensional homogeneity?

17.2 The ancient Greek view of dimensional homogeneity

Dimensional homogeneity is not unique to modern science. In various forms, it has been an integral part of science since the days of Aristotle.

The ancient Greek view of dimensional homogeneity is described by Drake (1974):

> *Algebraically, speed is now represented by a "ratio" of space traveled to time elapsed. For Euclid and Galileo, however, no true ratio could exist at all except between two magnitudes of the same kind.*

Similarly, Cohen (1985) states:

> *Aristotle and most early scientists, including Galileo, preferred to compare speeds to speeds, forces to forces, and resistances to resistances.*

The above passages indicate that, in the Greek view:

- Dimensions may *not* be added, subtracted, multiplied, or divided.
- Dimensioned quantities of identical dimension *may* be added, subtracted, and divided.
- Dimensioned quantities of different dimension may *not* be added, subtracted, multiplied, or divided.
- Equations may contain only numbers. Therefore they are inherently homogeneous.

The following by Galileo (1638) indicates that he did in fact share the ancient Greek view of homogeneity:

> *If two particles are carried at a uniform rate, the ratio of their speeds will be the product of the ratio of the distances traversed by the inverse ratio of the time-intervals occupied.*

Notice that:

- Each term in Galileo's verbal equation involves dividing a dimensioned quantity by a dimensioned quantity of identical dimension, as allowed in the Greek view.
- All terms are pure numbers, since each term has the same dimension in numerator and denominator, as required in the Greek view.
- Since all terms are pure numbers, the equation is homogeneous.

Notice that Galileo saw no conflict between the concept of speed and the view that distance cannot be divided by time. To Galileo, distance and time were necessary to quantify speed, but speed had nothing to do with *dividing* distance by time. Galileo divided distance by distance, and time by time, but he would *not* divide distance by time. In his view, dividing distance by time would be irrational and impossible.

The purpose of Galileo's verbal equation was to state that speed equals distance traversed divided by time-interval occupied. However, he would not state it in this simple form because he would not divide distance by time. The only reason his equation deals with two particles is so that he could describe speed without dividing distance by time.

The ancient Greek view prevailed for 2000 years. However, because it does not allow mathematical operations on dimensioned quantities of different dimension, it greatly complicates descriptions of Natural phenomena, as evidenced by Galileo's verbal equation.

17.3 Newton's view of dimensional homogeneity

Newton's view of homogeneity is described by Kroon (1971):

> *Newton did not concern himself with dimensions or units; he merely expressed proportionality according to the custom of his days.*

For example, Westfall (1993) states that Newton expressed his second law as follows:

> *The change of motion is proportional to the impressed force . . .*

Note that Newton's verbal equation is inhomogeneous, since the dimension of force differs from the dimension of motion.

17.4 Fourier's view of dimensional homogeneity

Maxwell (1873) states:

> *The theory of dimensions was first stated by Fourier, Theorie de Chaleur.*

Fourier describes his view of homogeneity in the following:

> *. . . the terms of (an) equation could not be compared, if they had not the same exponent of dimensions.*
>
> *. . . this (view of dimensional homogeneity) . . . is derived from primary notions on quantities; for which reason, in geometry and mechanics, it is the equivalent of the fundamental lemmas which the Greeks have left us without proof.*

Note that Fourier does not *prove* that homogeneity is required. He simple observes that "*it . . . is the equivalent of the fundamental lemmas (axioms)*" handed down from the ancient Greeks, and accepted without proof.

17.5 The conventional view of dimensional homogeneity

The conventional view of dimensional homogeneity was conceived by Fourier (1822). It is described in modern terminology by Langhaar (1951):

> *. . . an equation of the form $x = a + b + c + \ldots$ is dimensionally homogeneous if, and only if, the variables x, a, b, c, . . . all have the same dimension. . . . If a derived equation contains a sum or a difference of two terms that have different dimensions, a mistake has been made.*
>
> *. . . dimensions must not be assigned to numbers, for then any equation could be regarded as dimensionally homogeneous.*

White (1970) describes the conventional view in a similar way:

> *(Dimensional homogeneity) is almost a self-evident axiom in physics. . . (It) can be stated as follows:*
>
>> *If an equation truly expresses a proper relationship between variables in a physical process, it will be dimensionally homogeneous; ie each of its additive terms will have the same dimension.*

Implicit in the above is the view that dimensions may be multiplied or divided, but they may *not* be added or subtracted.

The conventional view may be summarized as follows:

- Dimensions *may* be multiplied and divided, but *not* added or subtracted.

- Dimensioned quantities of identical dimension *may* be added, subtracted, multiplied, and divided.

- Dimensioned quantities of different dimension *may* be multiplied and divided, but *not* added or subtracted.

- The equal sign indicates numerical equality *and* dimensional homogeneity.

17.6 The self-evident nature of the conventional view

The conventional view of homogeneity has been globally accepted for almost two hundred years, and it is learned very early in a technical curriculum. The end result is that, because the conventional view has been global for so long, and because it is generally learned at an age when one does not seriously question the material being taught, the conventional view has understandably come to be viewed as "almost a self-evident axiom in physics".

If the conventional view of homogeneity were in fact "almost a self-evident axiom", it would be quite easy to demonstrate its validity. Recall that even Fourier, the pioneer of the conventional view, was unable to demonstrate that his view of homogeneity was anything more than an arbitrary and questionable point of view. Fourier simply offered his view without proof, stating "*it . . is the equivalent of the fundamental lemmas which the Greeks have left us without proof*", and therefore seemingly above reproach.

It should also be noted that Galileo and Newton were physicists of the first rank, and neither subscribed to the conventional view of homogeneity. If the conventional view were in fact "almost a self-evident axiom in physics", it would certainly have been the view held by Galileo and Newton.

17.7 The inhomogeneous behavior of engineering phenomena

Engineering phenomena are cause-and-effect processes, and the effect generally has different dimensions than the cause. Therefore engineering phenomena generally exhibit *in*homogeneous behavior. For example:

- An emf causes an electric current. Since the dimension of emf differs from the dimension of electric current, electrical phenomena are inhomogeneous.

- A temperature difference causes a heat flux. Since the dimension of temperature differs from the dimension of heat flux, heat flow phenomena are inhomogeneous.

- A stress causes a strain. Since the dimension of stress differs from the dimension of strain, stress/strain phenomena are inhomogeneous.

- A pressure difference causes a flow rate. Since the dimension of pressure differs from the dimension of flow rate, fluid flow phenomena are inhomogeneous.

In the conventional view, engineering phenomena are rigorously described only by equations that are homogeneous. However, the reality is that engineering phenomena generally exhibit behavior that is *in*homogeneous.

17.8 Homogenizing inhomogeneous, *proportional* behavior

The manner in which Hooke's law (1676) is homogenized exemplifies the manner in which the creation of parameters such as *R, h, E* makes it possible to transform inhomogeneous, *proportional* expressions to homogeneous equations.

Based on Hooke's stress/strain data, it was concluded that Expression (17-1) describes the behavior of many materials over a considerable range:

$$\sigma \alpha \varepsilon \qquad (17\text{-}1)$$

Expression (17-1) is generally referred to as Hooke's law.

Hooke's law is inhomogeneous, since the dimensions of stress and strain are not identical. Therefore, in Langhaar's words, "a mistake has been made". And in White's words, Hooke's law does not "truly express a proper relationship between" stress and strain.

In accordance with the conventional view, it is necessary to homogenize the inhomogeneous Hooke's law. It is homogenized by creating and introducing *E* in the following manner:

- Convert Hooke's law to an equation by introducing an arbitrary constant.

- Assume/postulate/theorize that the arbitrary constant in the equation is *not* a number. It is a dimensioned parameter.

- Assign a name and a symbol to the dimensioned parameter that is really a number. Assign it the name "modulus", and the symbol *E*.

- Note that the equation would be homogeneous if the dimensioned parameter that is really a number had the necessary dimensions.

- *Make* the equation homogeneous by arbitrarily assigning the necessary dimensions to the dimensioned parameter that is really a number.

The homogeneous equation obtained by creating *E* is known as Young's law, Eq. (17-2).

$$\sigma = E\,\varepsilon \qquad (17\text{-}2)$$

Note that *E* is the ratio σ/ε. Also note that this ratio is constant if σ is proportional to ε, and is variable if σ is not proportional to ε.

17.9 A critical appraisal of the homogenization process

Note the following in the homogenization of Hooke's law:

- The process is flawed because it postulates that an arbitrary constant is *not* a number—it is a dimensioned parameter.

- The process is flawed because it "validates" the presumption of homogeneity by arbitrarily assigning the dimension required for homogeneity to a parameter that is really a number.

- It is not rational to assign dimensions to numbers. Doing so violates the ground rule stated by Langhaar: "*dimensions must not be assigned to numbers, for then any equation could be regarded as dimensionally homogeneous*".

- The homogeneity of Young's law does not alter the fact that the phenomenon it purports to describe exhibits behavior that is inhomogeneous.

17.10 Fourier's homogenization methodology

The homogenization methodology described above was pioneered by Fourier (1822) who used it to homogenize heat flow behavior he had observed.

Based on experiment, Fourier concluded that:

$$q_{cond} \; \alpha \; dT/dx \qquad (17\text{-}3)$$

$$q_{conv} \; \alpha \; \Delta T \qquad (17\text{-}4)$$

Because of his view of dimensional homogeneity, Fourier wished to describe the inhomogeneous behavior of Expressions (17-3) and (17-4) by homogeneous equations. In order to attain homogeneity, Fourier:

- Converted the above expressions to equations by introducing an arbitrary constant into each equation.
- Postulated that the arbitrary constants are really parameters.
- Assigned the symbols k and h to the parameters that are really arbitrary constants.
- Assigned to k and h those dimensions that would result in homogeneous equations. Equations (17-5) and (17-6) are the result.

$$q_{cond} = k\{T\}(\, dT/dx) \qquad (17\text{-}5)$$

$$q_{conv} = h \; \Delta T \qquad (17\text{-}6)$$

Notice that:

- Both expressions are homogeneous, as desired.
- k is the ratio $q_{cond}/(dT/dx)$. This ratio is currently considered to be constant for all practical materials.
- h is the ratio $q_{conv}/\Delta T$. This ratio is currently considered to be constant for some heat transfer phenomena, and variable for others.

17.11 Why Fourier's contemporaries converted to his view

Fourier's contemporaries converted to his view of homogeneity because he solved many problems that his contemporaries were unable to solve. He attributed his success to the fact that his equations were homogeneous, and he attributed the failure of his contemporaries to inhomogeneity in their equations.

For example, Biot (1804) presented the results of an experiment in heat conduction. He generalized that, six feet from the heated end of *any* solid iron bar, the temperature of the bar would be less than one degree above ambient because the heated end of the bar would melt before the temperature six feet from the heated end reached one degree.

Fourier (1822) pointed out that Biot's result was in error because his equations did not consider the effect of the cross section of the bar, and therefore his equations were inhomogeneous. In reference to Biot's erroneous conclusion, Fourier states:

> *If we did not make a complete analysis of the elements of the problem, we should obtain an equation not homogeneous and, a fortiori, we should not be able to form the equations which express the movement of heat in more complex cases.*

In short, Biot's conclusion was incorrect because the equations on which he based his conclusion were inhomogeneous.

Fourier's argument is false. The cause of Biot's error was not that his equations were inhomogeneous, but rather that his equations omitted an important parameter. When important parameters are missing, equations are necessarily incorrect, whether they are homogeneous or inhomogeneous.

17.12 Homogenizing inhomogeneous, *nonlinear* behavior

The homogenization of inhomogeneous, *proportional* behavior is relatively straightforward. Proportional equations contain only one arbitrary constant, and both variables have an exponent of unity. Homogenization requires merely that the proportionality constant be assigned the status of a parameter and the dimension of the ratio of the primary parameters.

The homogenization of inhomogeneous, *nonlinear* behavior is less straightforward. For example, experiments in heat transfer by natural convection indicate that

$$q \propto \Delta T^n \qquad (17\text{-}7)$$

where the value of *n* depends on geometry. Typical values are 1.2, 1.25, 1.33, and 1.35.

Using the method Fourier pioneered, the homogenization of Expression (17-7) requires that the arbitrary constant in the resultant equation be assigned a unique symbol and the dimension $q/\Delta T^n$.

Since *n* has several values, the proportionality constant must be assigned a unique symbol and different dimensions for each value of *n*. The end result is that several heat transfer coefficients are required to describe natural convection heat transfer, and each one requires a unique symbol and a different dimension. Such a system is obviously unsatisfactory.

In conventional engineering, the problem of homogenizing nonlinear behavior is usually avoided by correlating dimensionless groups. Since dimensionless groups are pure numbers, homogeneity is assured.

For example, correlations in the form $N_{Nu}\{N_{Gr}\}$ contain only pure numbers, and therefore homogeneity is assured,

17.13 The rationale of multiplying and dividing dimensions

Fourier (1822) pioneered the view that it is rational to multiply and divide dimensions. He stated:

> . . . *every undetermined magnitude or constant has one dimension proper to itself, and the terms of one and the same equation could not be compared, if they had not the same exponent of dimensions. We have introduced this consideration into the theory of heat* . . .

In other words, dimensions must be multiplied and divided in order to verify that all terms in an equation have the same "exponent of dimension".

Fourier's view is not rational because dimensions are *things*, and mathematical operations can *not* be performed on *things*. They can be performed only on numbers and numbers of things. For example, horses can *not* be divided by fields because horses and fields are *things*. But the *number* of horses may be divided by the *number* of fields devoted to grazing in order to determine the average *number* of horses grazing in each field.

Because dimensions are *things*, they can *not* be multiplied or divided. For example, pounds cannot be divided by seconds. If pounds could be divided by seconds, it would be possible to answer the question "How many times does a second go into in a pound?" Thus Fourier's claim that

> *the terms of one and the same equation could not be compared, if they had not the same exponent of dimensions*

is not rational because it requires that dimensions be multiplied and divided to verify that all terms have the same "*exponent of dimension*".

Therefore Fourier's view of homogeneity—the conventional view—is not rational, and should be abandoned.

17.14 Summary

- The conventional view of homogeneity is not logical because it requires that the inherently *inhomogeneous* behavior of engineering phenomena be described by equations that are *homogeneous*.

- In the conventional view of homogeneity, dimensions may be multiplied and divided. This view is not rational because dimensions can *not* be multiplied and divided.

- Fourier's homogenization method is accomplished by assigning dimensions to numbers. This violates a ground rule of dimensional homogeneity because assigning dimensions to numbers allows any equation to be regarded as dimensionally homogeneous.

- Fourier's homogenization method is based on preserving homogeneity by creating "parameters" such as *R, h,* and E. These "parameters" are undesirable because they are in fact ratios of the primary parameters, and they complicate the solution of nonlinear problems by making it necessary to solve problems with the variables combined.

Chapter 18

Dimensional homogeneity in the new engineering

18 Introduction

This chapter describes the new engineering view of dimensional homogeneity. The principal advantage of the new engineering view is that it results in a simpler engineering science because it allows the primary parameters to remain separate, whereas they are combined in conventional engineering.

18.1 The new engineering view

The new engineering view of homogeneity is derived from the view presented in Adiutori (1992). It is reflected in the following:

- Mathematical operations may be performed *only* on numbers—pure numbers, and numbers of things.

- When mathematical operations are performed on numbers of things, the mathematical operations are performed *only* on the numbers. For example, if 12 gallons of water flows in 3 minutes, the 12 is divided by the 4 to result in an average flow rate of 4 gallons per minute, but no mathematical operation is performed on "gallons" or "minutes".

- Mathematical operations can *not* be performed on dimensions. For example, gallons can *not* be divided by minutes. If gallons could be divided by minutes, it would be possible to answer the question "How many times does a minute go into a gallon?"

- Since mathematical operations may be performed only on numbers, and since equations inherently involve mathematical operations, equations may contain only numbers.

- Since equations may contain only numbers, symbols in equations must represent numerical values of parameters in specified dimensions.

- Since equations may contain only numbers, the equal sign indicates numerical equality, but makes no statement about homogeneity. None is required, since equations are inherently homogeneous.

- Engineering phenomena exhibit inhomogeneous behavior. However, there is no reason to suppose that engineering phenomena are rigorously described only by equations that are also inhomogeneous.

18.2 The impact of the new engineering view

The new engineering view of homogeneity has the following direct impact on methodology:

- Symbols represent the numerical values of parameters in specified dimensions. (In conventional engineering, symbols represent parameters).

- Because symbols represent numerical values, equations are inherently homogeneous. (In conventional engineering, homogeneity is required for rigor, but equations are not inherently homogeneous.)

- Since equations are inherently homogeneous, parameters such as *R, h, E* are unnecessary because their sole purpose is to homogenize proportional expressions such as Hooke's law.

- Parameters such as *R, h, E* are undesirable because they combine the primary parameters, and this greatly complicates the solution of nonlinear problems.

- Because parameters such as *R, h, E* are unnecessary and undesirable, they are abandoned. Engineering phenomena are described and problems are solved with the primary parameters *separated*.

- Writers of equations must ensure that equations are numerically and dimensionally correct when the specified dimensions are used. Users of equations must be careful to use the symbol dimensions specified by the writer of the equation.

18.3 Dimension symbolism in the new engineering

The dimension symbolism in conventional engineering denotes the multiplication and division of dimensions. The conventional symbolism is misleading when used with the new view of homogeneity, since mathematical operations are not performed on dimensions.

For example, in the conventional view, the symbolism "ft/sec" indicates the dimension ft divided by the dimension sec. The symbolism "lbs/ft^3" indicates the dimension lbs divided by the dimension ft cubed.

In the new engineering, ft is never divided by sec, and therefore it would be misleading to use "ft/sec" when velocity is quantified using ft and sec. Similarly, lbs is never divided by ft, and ft is never multiplied by ft, and therefore it would be misleading to use "lbs/ft^3" when density is quantified using lbs and ft.

A more appropriate dimension symbolism is to simply note the dimensions used to quantify the parameter, as in the following table:

Conventional symbolism	**New symbolism**
velocity = 20 ft/sec	velocity = 20 ft,sec
density = 62 lbs/ft^3	density = 62 lb,ft
heat flux = 87 Btu/hrft^2	heat flux = 87 Btu,hr,ft

"Velocity = 20 ft,sec" indicates that the numerical value of velocity is 20 when quantified by the dimensions ft and sec. "Density = 62 lb,ft" indicates that the numerical value of density is 62 when quantified by the dimensions lb and ft.

Although the conventional symbolism is misleading, it makes no practical difference which dimension symbolism is used.

18.4 Hooke, Newton, and Galileo

Hooke's law is generally considered to be:

> Stress is proportional to strain.

However, there can be no doubt that Hooke's intended meaning was

> The *numerical value* of stress (in arbitrary dimensions) is proportional to the *numerical value* of strain.

Hooke induced the law from the numerical values of the stress and strain data he had obtained. Surely he recognized that it is not stress that is proportional to strain—it is the *numerical value* of stress that is proportional to the *numerical value* of strain. And he reasonably and correctly assumed it would be understood that the law refers to the *numerical values* of stress and strain.

Everyone who is aware of Hooke's law understands that it means the *numerical value* of stress is proportional to the *numerical value* of strain, even though it is stated in the form "stress is proportional to strain".

The reality is that:

- Hooke's law, as he intended it, concerns *numerical values* of stress and strain.
- Hooke's law, as it is generally understood, concerns *numerical values* of stress and strain.
- Because Hooke's law concerns *numerical values* of stress and strain, it is *inherently homogeneous*.
- Because Hooke's law is inherently homogeneous, there is no need to homogenize it by creating and introducing *E*.

Similarly, recall Newton's statement:

> *The change of motion is proportional to the impressed force* . . .

There can be no doubt that Newton's intended meaning was

> The *numerical value* of the change of motion (in arbitrary dimensions) is proportional to the *numerical value* of the impressed force (in arbitrary dimensions).

It seems quite certain that Hooke and Newton and their contemporaries did not concern themselves with dimensions because they reasonably and correctly assumed it would be understood that proportional expressions refer to *numerical values* of parameters in arbitrary dimensions.

It also seems quite certain that Hooke and Newton and their contemporaries agreed with the early Greeks and with Galileo (and with the new engineering view) that mathematical operations can *not* be performed on dimensions, and that equations may contain *only* numbers.

18.5 The principal advantage of the new engineering view

Relative to the conventional view of homogeneity, the principal advantage of the new engineering view is that it results in a simpler science of engineering because it allows the behavior of engineering phenomena to be described in a simple way, and with the primary parameters *separated*.

The conventional view also allows the behavior of engineering phenomena to be described in a simple way, but requires the creation of "parameters" such as R, h, and E. These parameters are undesirable because they combine the primary parameters. This makes it necessary to solve problems with the variables combined, and greatly complicates the solution of nonlinear problems.

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